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User Guide to the Aircraft Cumulative Probability Chart Template

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Defence Science and Technology Organisation

***AeroStructures Technologies**

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ABSTRACT

To ensure aircraft structural integrity is maintained to an acceptable level, probabilistic approaches may be used to calculate the risk of cracking (or failure) over the life of the aircraft or fleet. One such risk analysis technique employs a lognormal probability distribution to model the likelihood of cracking (or failure) in the fleet with respect to hours. This technique was programmed into Microsoft Excel to create a simple and easy to use template. An outline of the theory behind the probabilistic approach is provided as well as a comprehensive user guide to the template. This template allows the quick and simple determination of probability distributions of cracking (or failure) which may be used to assess the life of aircraft structures.

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User Guide to the Aircraft Cumulative Probability Chart Template

Executive Summary

To ensure aircraft structural integrity is maintained to an acceptable level, probabilistic approaches may be used to calculate the risk of cracking (or failure) over the life of the aircraft or fleet. Traditionally, aircraft structures have been lifed using a safe life or damage tolerance approach. However, a probabilistic approach may also be employed and is particularly useful when including the results from fleet inspections or where the more traditional techniques prove inadequate. This type of approach was used as part of the management stategy for the RAAF P-3C Orion fleet.

One such probability approach employs a lognormal probability distribution to model the likelihood of cracking (or failure) in the fleet with respect to hours. This technique was programmed into Microsoft Excel to create a simple and easy to use template. The template may be used to calculate the probability of cracking (or failure) at a given number of hours or the number of hours associated with a given probability of cracking (or failure). An outline of the theory behind the probabilistic approach is provided as well as a comprehensive user guide to the template.

A Microsoft Excel template has been developed which allows the determination of probability distributions of cracking (or failure). A user guide to the template has also been provided. This template may be used to quickly and simply assess the life of aircraft structures using a probabilistic approach. This template was used in assessing the fleet demonstrated life and probability of failure for locations on the RAAF P-3C Orion Fleet, and was integrated into the P-3C Structural Management Plan.

Authors

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Mr Murtagh graduated from RMIT with a Bachelor of Aerospace Engineering (Honours) in 1996 and joined AeroStructures in June 1997. He spent three years supporting the P-3 at the RAAF Edinburgh office before returning to the Melbourne office. Since then, he has conducted work for both DSTO and the ADF in support of the P-3 and other ADF aircraft in the fields of composites, usage, fatigue and damage tolerance. From September 2005 until May 2007, he supported the DSTO SLAP program. In May 2007, Mr Murtagh joined Marshall Aerospace, providing ASIP Management for the Canadian C-130 Fleet.

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Mr Maxfield graduated from RMIT in 2004 with a Bachelor of Aerospace Engineering (Honours). In August 2005 he commenced work with AeroStructures Technologies at DSTO. During this time he had been providing Engineering Support to the Air Vehicles Division of DSTO, primarily on the RAAF P-3 SLAP program. In January 2007, Mr Maxfield joined Air Vehicles Divion at DSTO, working on Airworthiness Standards and the P-3 SLAP Program. Since this time, he has also worked as test Engineer for the F/A-18 FINAL Testing. Mr Maxfield is currently working on local support for the RAAF C-130J-30 Full Scale Fatigue Test Program.

Contents

List of Abbreviations

AFHRS	Airframe Hours
CDF	Cumulative Distribution Function
BuNo	Bureau Number
DSTO	Defence Science and technology Organisation
DTA	Damage Tolerance Analysis
FI	Fatigue Index
ID	Identity
No	Number
Prob	Probability
RAAF	Royal Australian Air Force
Ref	Reference
SD	Standard Deviation
USN	United States Navy
VBA	Visual Basic for Applications

1. Introduction

To ensure aircraft structural integrity is maintained to an acceptable level, probabilistic approaches are often used to calculate the risk of cracking (or failure) over the life of the aircraft or fleet. This risk analysis may then be used to help substantiate changes in the management strategy of an aircraft or component. For example, a risk analysis may be used to demonstrate that an increase in inspection threshold Airframe Hours (AFHRS) is acceptable with respect to the risk of cracking.

One risk analysis technique employs a probability distribution to model the likelihood of cracking (or failure) in the fleet with respect to AFHRS (or other relevant metric). Known cracking (or failure) data points obtained from fleet aircraft or a test article are used to define this probability distribution. The probability distribution can then be used to determine the probability of cracking at a given AFHRS, or the AFHRS corresponding to a defined probability of cracking.

A method for determining the likelihood of cracking and failure was developed initially for assessing the fleet demonstrated life based on United States Navy (USN) inspection data for the Royal Australian Air Force (RAAF) P-3C Orion. The method utilised the statistical method originally described in Reference 2 and used in Reference 5 for intermediate P-3 life substantiation.

To enable the quick and simple definition of such probability distributions for multiple aircraft structural locations, a standardised system for data entry and output is required. A template was therefore developed in Microsoft Excel to perform this task.

This template will:

- define a lognormal probability distribution for cracking in Fleet 'A' based on the input data points
- define a lognormal probability distribution for cracking in a related fleet, Fleet 'B', based on the relative severity of the two fleets at the structural location of interest
- define a probability distribution for failure in Fleet 'B', based on the lognormal probability distribution for cracking and the number of AFHRS for a crack to grow to failure (critical crack length)
- calculate the AFHRS corresponding to a user specified level of cumulative probability of cracking or failure (user option)
- calculate a cumulative probability of cracking or failure (user option) at a user specified AFHRS
- calculate the Fatigue Index (FI) at which inspections should begin, based upon the AFHRS corresponding to the user specified cumulative probability of cracking or failure (user option) and the fleet average FI rate (per 1,000 AFHRS)

This template was utilised as part of the Structural Management Plan for the RAAF P-3C Orion [3] and further utilised in the follow up spanwise splice analysis [4].

This User Guide presents information regarding this template including assumptions, layout, data entry, output data and the underlying calculations. Guidance regarding manipulation and use of aircraft cracking data for use in this template is also presented.

The Excel template may be obtained by contacting by letter, Chief, Air Vehicles Division, DSTO, Fishermans Bend, Victoria 3207.

2. How to Use this Guide

This guide is intended for users interested in using or modifying the template. Although basic information regarding risk analysis approaches, methods and applications (tools) are presented, they are not comprehensive. It is the user's responsibility to become familiar with and understand these approaches, methods and applications (tools) before use of the template, including any implicit assumptions.

The remainder of this guide has been broken into six Chapters:

- Fundamentals of Lognormal Probability Distributions
- Probability Distribution Input Data
- The Cumulative Probability Distribution Template
- The Cumulative Probability Distribution Worksheet for Summary Data
- The Cumulative Probability Distribution Worksheet for Detailed Data
- Template Troubleshooting

The Chapter titled "Fundamentals of Lognormal Probability Distributions" presents basic background information regarding lognormal probability distributions and its application to fatigue analysis.

The Chapter titled "Probability Distribution Input Data" presents information regarding the collation and treatment of input data required for definition of probability distributions.

The Chapter titled "The Cumulative Probability Distribution Template" presents a brief overview of the Cumulative Probability Distribution Template.

The Chapter titled "The Cumulative Probability Worksheet for Summary Data" presents details regarding the worksheet to be used to define cumulative probability distributions if only a summary of the cracking (or failure) data is available. As a minimum, summary data comprises average AFHRS of cracked aircraft, the number of aircraft cracked and the number of aircraft in the fleet.

The Chapter titled “The Cumulative Probability Worksheet for Detailed Data” presents details regarding the worksheet to be used to define cumulative probability distributions if detailed data regarding cracking (or failure) in fleet is available. As a minimum, detailed data includes the following information for each aircraft included in the analysis: tail number, AFHRS, and whether it is cracked (or failed).

The Chapter titled “Template Troubleshooting” presents basic troubleshooting information pertaining to common issues or questions related to the template.

3. Fundamentals of Lognormal Probability Distributions

For a given fleet, the occurrence of cracking will typically be distributed over a range of AFHRS. However, assuming each aircraft in the fleet is used in a similar manner, it can be expected that the cracking will be grouped around a mean AFHRS.

Historical data indicates that the distribution of cracking around this mean AFHRS can often be modelled using a lognormal probability distribution. Other distributions, including the Weibull distribution, may also be appropriate but are not addressed here. An example of a lognormal probability distribution is presented in Figure 1. The probability of an aircraft cracking at a particular AFHRS may be obtained from this distribution.

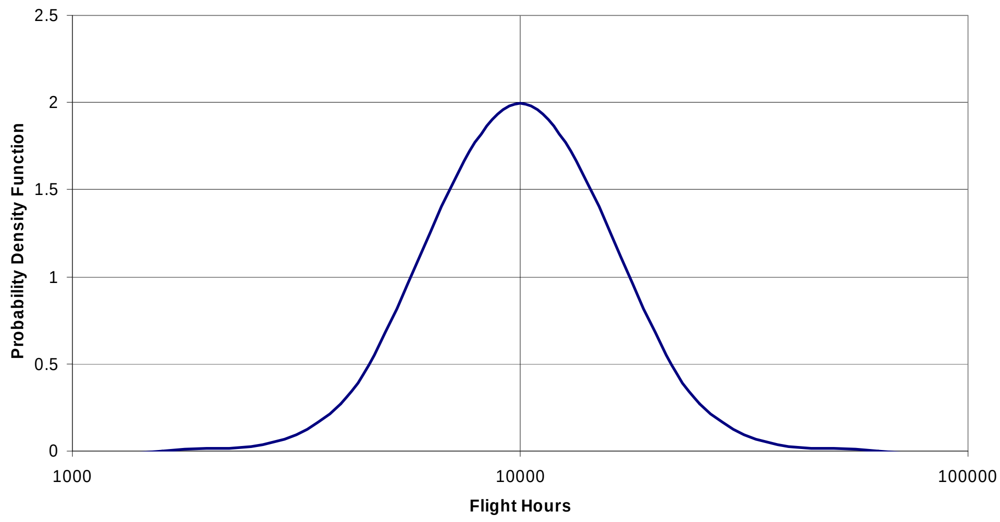

Figure 1: Typical Lognormal Probability Distribution

Important aspects of this lognormal distribution are:

- the mean AFHRS has the highest probability,
- the distribution is symmetrical about the mean,
- the area under this curve is equal to 1.

Only two variables are required to define a lognormal probability distribution: the mean AFHRS and the standard deviation. The standard deviation is a measure of how spread out the data is. If the data is bunched close together, the probability curve in Figure 1 becomes very narrow and tall, and the standard deviation is small. However, if the data is spread out, the probability curve in Figure 1 becomes very wide and flat, and the standard deviation is large.

However, it is usually more useful to know the probability of an aircraft cracking at or before a particular AFHRS. This probability is called the cumulative probability, and is calculated as the area under the lognormal probability curve to the left of the particular AFHRS, as shown in Figure 2.

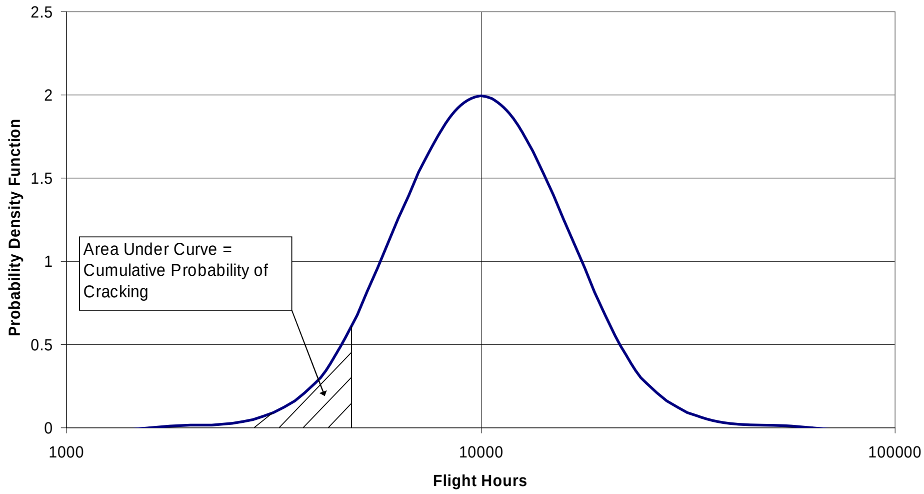

Figure 2: Calculation of the Lognormal Cumulative Probability of Cracking

The area under this curve can be defined by the following equation:

$$\mathrm{CDF:F(T)} = \int_0^T \frac{1}{\sigma t\sqrt{2\pi}} e^{-\left(\frac{1}{2\sigma^2}\right)(\ln t - \ln T_{50})^2} dt$$

where CDF is the cumulative distribution function, F(T) is a function of time, σ is the standard deviation of the probability distribution, t is the time and T_{50} is the mean of the probability distribution.

Therefore, for AFHRS significantly below the mean, the lognormal cumulative probability of cracking is very low. Similarly, for AFHRS significantly above the mean, the lognormal cumulative probability of cracking is very high. The lognormal cumulative probability of cracking is equal to 0.5 (50%) at the mean AFHRS.

When the lognormal cumulative probability is plotted against AFHRS on a special lognormal probability chart, it forms a straight line. An example of a lognormal cumulative probability chart is presented in Figure 3.

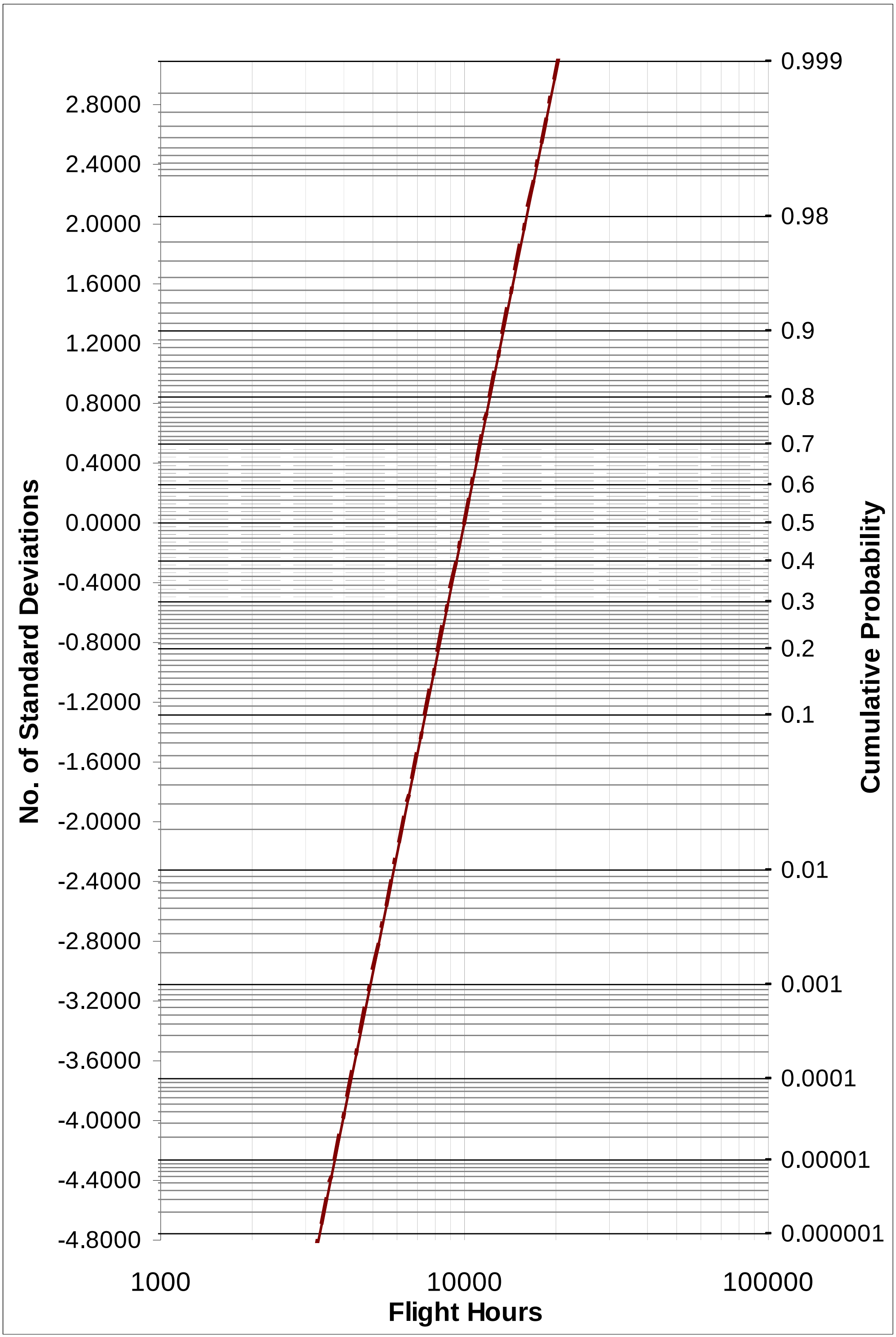

Figure 3: The Lognormal Cumulative Probability Chart

The data (nominally cracking and AFHRS) in the lognormal cumulative probability chart is presented on a log scale. The lognormal cumulative probability is also plotted on a log scale (right vertical axis), however it is centred around a cumulative probability of 0.5, the cumulative probability of the mean AFHRS.

A second vertical scale (left) is presented in Figure 3 titled "Number of Standard Deviations". This is a linear scale, hence the number of standard deviations from the mean is linearly proportional to the log(AFHRS). Therefore, the standard deviation can be calculated from the slope of the lognormal cumulative probability distribution. It is equal to the difference in log(AFHRS) divided by the difference in number of standard deviations from the mean.

Armed with this information, if the mean and standard deviation of a lognormal probability distribution are known, the lognormal cumulative probability distribution can be plotted on the lognormal cumulative probability chart. It is then possible to read the cumulative probability of cracking for any given AFHRS directly off the chart.

If the standard deviation is not known (but the mean is), it may be possible to assume a value for the standard deviation based on historical data. A number of airworthiness standards give guidance on acceptable standard deviations for use in fatigue analysis. These include DEF STAN 00-970 [1], which suggests a standard deviation of 0.11 is acceptable.

If the mean is not known (the standard deviation is assumed), it is possible to generate the lognormal probability distribution if a single data point on the distribution is known. For example, if one aircraft in a fleet of four cracked at 5,000 AFHRS, the cumulative probability of cracking is 0.25 (1/4) at 5,000 AFHRS. The lognormal cumulative probability distribution generated from this data point (assuming a standard deviation of 0.1) is presented in Figure 4. The mean AFHRS (where the trendline crosses a cumulative probability equal to 0.5) can be read directly from the chart.

As an alternative to the graphical approach used to calculate the mean of the lognormal cumulative probability distribution (standard deviation is assumed), the mean can be calculated using statistical tables or functions (such as NORMDIST in Excel).

In the previous example, the cumulative probability was equal to 0.25 (1/4). This is called a ranked cumulative probability. If there were two aircraft cracked, at 5,000 and 6,000 AFHRS respectively, then the first aircraft would have a ranked cumulative probability of 0.25 (1/4). The second aircraft would have a ranked cumulative probability of 0.5 (2/4), since there were two aircraft cracked at or below 6,000 AFHRS. It is implicitly assumed that all uncracked aircraft will crack at a higher AFHRS than the cracked aircraft.

It is possible to plot both of these data points on a cumulative probability chart, draw a straight line through them, and declare that line the lognormal cumulative probability of cracking distribution. However, this approach does not consider the possibility of scatter in the data. As a result, the calculated standard deviation may be significantly different to the true standard deviation.

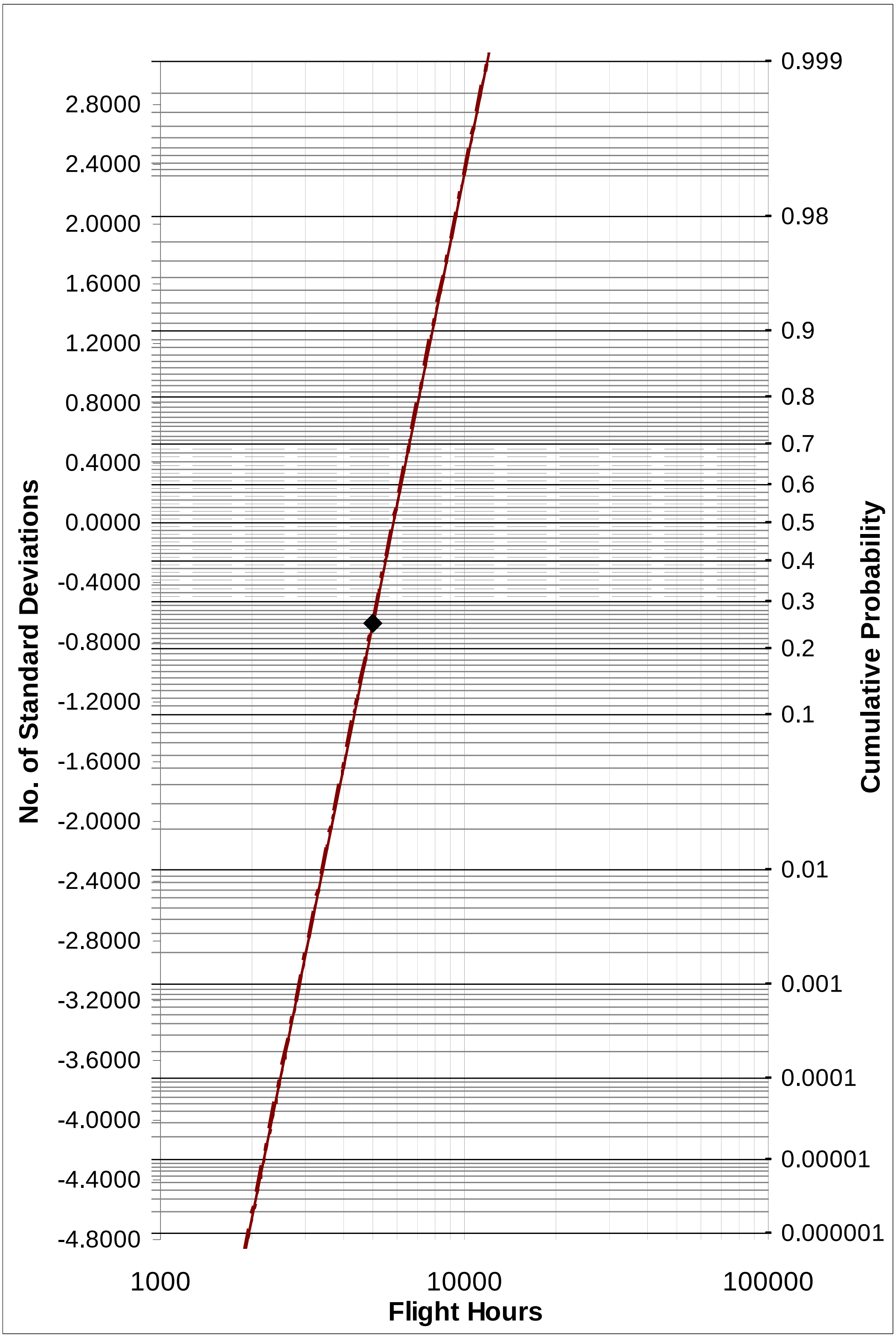

Figure 4: Lognormal Cumulative Probability Distribution Based on Single Data Point

As an alternative, the two data points may be averaged, thus generating a single data point that can be used with the assumed standard deviation (based on historical data). The AFHRS of the averaged data point is the logarithmic average. Thus the average data point of the above example would be located at 5,477 AFHRS (the log average of 5,000 and 6,000 AFHRS). The cumulative probability of cracking of the averaged data point would be 0.375 (linear average of 0.25 (1/4) and 0.5 (2/4)). This example is presented in Figure 5.

Multiple data points can be averaged using this approach. An equation that defines the average cumulative probability of cracking for multiple cracked aircraft (where the number of aircraft cracked is less than the number of aircraft in the fleet) is given as:

$$\text{Average Cumulative Probability} = \frac{Number\_of\_Aircraft\_Cracked + 1}{2 \times Number\_of\_Aircraft\_in\_Fleet}$$

If the number of aircraft cracked is equal to the number of aircraft in the fleet, then the average cumulative probability should be set to 0.5.

It will be noticed that if only one aircraft is cracked, then the average cumulative probability is equal to the percentage of aircraft cracked. However, if more than one aircraft is cracked, this is not the case. As the number of aircraft in the fleet increases, the average cumulative probability asymptotes toward a value equal to half the percentage of aircraft cracked. For example, if there are 40 aircraft in the fleet with 20 cracked, then 50% of the aircraft are cracked. The average cumulative probability is 0.263, which is close to 0.25 (0.5/2). If there were only 4 aircraft in the fleet, with two cracked, then the average cumulative probability would be 0.375, significantly more than 0.25.

The reason for this is that the average cumulative probability is equal to the average of the highest and lowest ranked cumulative probabilities. As the number of aircraft in the fleet increases, the lowest ranked cumulative probability (equal to 1/Number of aircraft in fleet) asymptotes toward zero, and therefore has little effect on the result.

Therefore, if an average AFHRS of cracking in the fleet is known, but actual fleet numbers and aircraft cracked are not, the average cumulative probability of failure may be estimated from the percentage of aircraft cracked (if this is known). However, this will lead to unconservative results, with the magnitude of the error dependent upon fleet size. The larger the fleet population, the smaller this error will be.

If a significant number of cracking data points are available, the data should be ranked by AFHRS and then the cumulative probability of each data point should be calculated and plotted on a lognormal cumulative probability chart. A trendline may then be fitted to the data to establish both the mean (AFHRS where the trendline crosses the lognormal cumulative probability of 0.5) and the standard deviation (derived from the slope of the trendline). An example of such a plot is presented in Figure 6. Note that the filled triangle in Figure 6 represents the average data point.

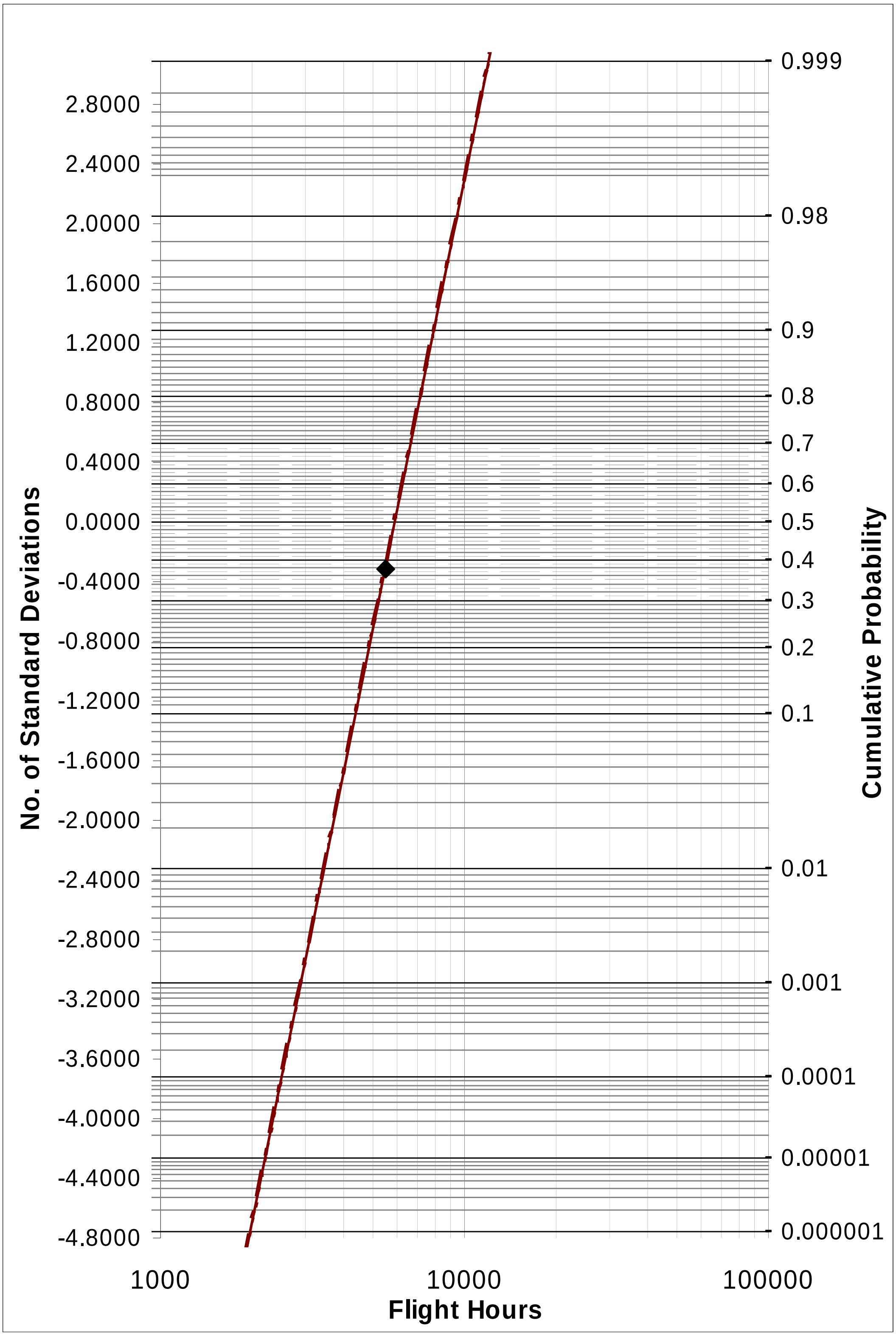

Figure 5: Lognormal Cumulative Probability Distribution Based on an Averaged Data Point

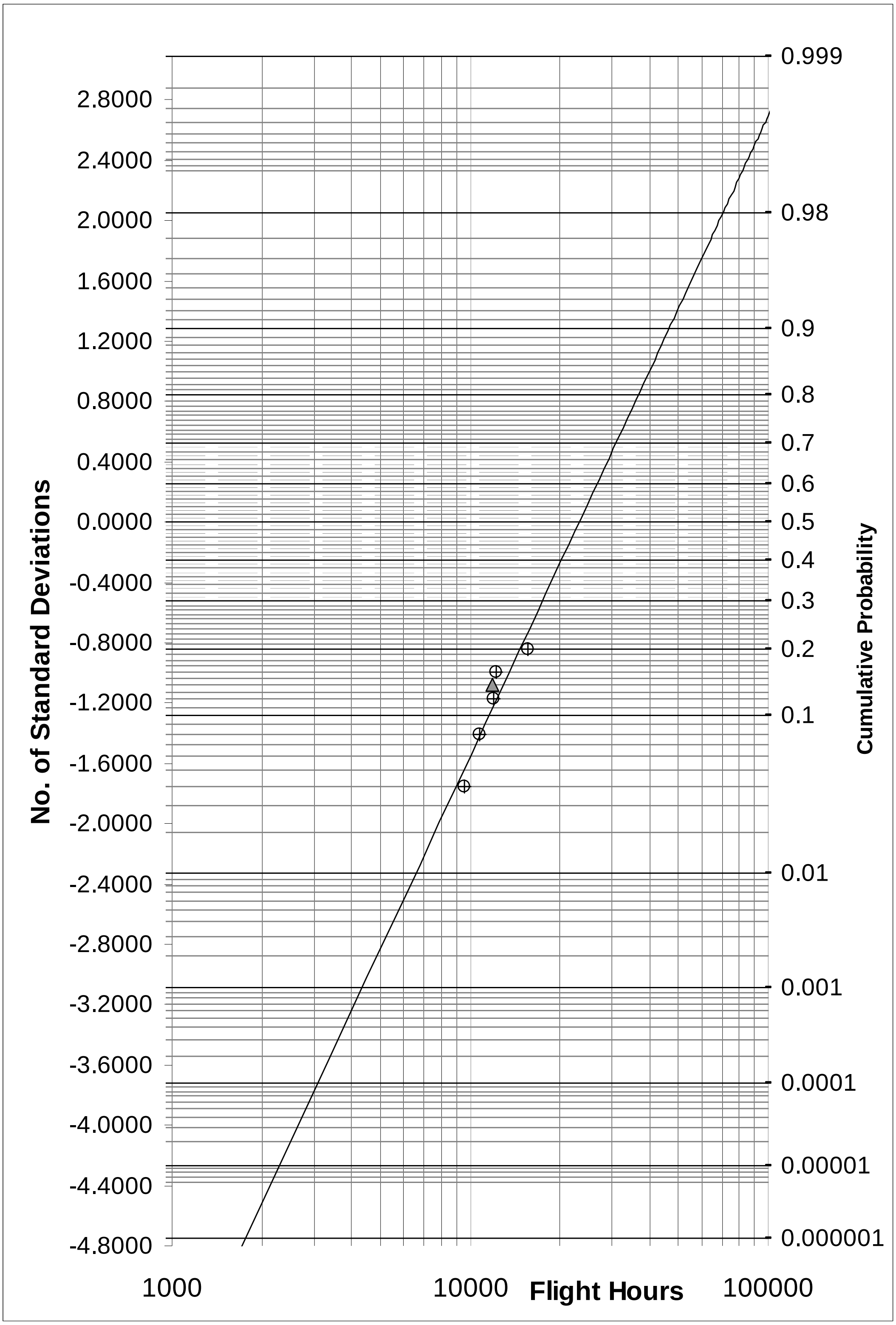

Figure 6: Lognormal Cumulative Probability Distribution for Multiple Cracks

The ranked cumulative probability of each data point is calculated by:

- sorting all of the cracked aircraft by AFHRS (lowest to highest);
- allocating a rank to each cracked aircraft such that rank, i, of the aircraft with the lowest AFHRS is equal to 1 and the rank of cracked aircraft with the highest AFHRS is equal to the number of cracked aircraft;
- the ranked cumulative probability for each cracked aircraft is then equal to its rank, i, divided by the number of aircraft in the fleet.

If all aircraft are cracked, then the highest rank will be 1, and this should be adjusted to equal 0.9999 to prevent errors in the Excel NORMDIST function.

Like all trendline analysis, care must be taken to ensure that the trendline is not adversely influenced by outliers, particularly those which may not be representative of the fleet.

3.1 Analysis (method) Assumptions

The fleet data used follows a lognormal distribution against AFHRS. This lognormal distribution applies to small crack growth, often referred to as crack initiation. Crack growth is considered to be a fixed time for all cracks at the analysis location. Therefore, due to the lognormal assumption for crack initiation, both probability of cracking and probability of failure analysis are considered to be lognormal.

The fleet data used is a representative sample of the fleet in terms of mean and scatter. It does not represent a skewed sample due to usage or age. If it does represent the older aircraft in the fleet, these aircraft are representative of the rest of the fleet when they reach the equivalent age.

The scatter in fleet B is similar to the scatter in fleet A, and therefore the same standard deviation is applicable for both fleets.

4. Probability Distribution Input Data

4.1 Introduction

This Chapter presents information regarding the collation and treatment of input data required for the definition of probability distributions.

4.2 Important Definitions

4.2.1 Cracking

Cracking is defined as the occurrence of a crack at the analysis location. For accuracy, it is important that all cracking data pertains to a crack of a nominal fixed length. If the cracking

data pertains to various crack lengths, it may be necessary to adjust the data before use as input for the definition of a probability distribution (refer Chapter 4.7).

4.2.2 Failure

Unless otherwise stated, failure is defined as the point at which a crack at the analysis location reaches the critical crack length and failure of the part is assumed (ie no load carrying capacity). If cracking (at a nominal crack length) occurs at "X" AFHRS, and the analytical crack growth from the nominal crack length to the critical crack length is "Y" AFHRS, then failure is assumed to occur at "X+Y" AFHRS. In many cases, due to fail-safe design, failure of the part will not cause loss of the aircraft, however, due to the risks, this has been defined as the failure criteria.

4.2.3 Fleet

A fleet is defined as a collection of aircraft of the same (or similar) structural configuration that are subject to similar usage. Cracking in aircraft of different structural configuration or usage may not be representative of the fleet and thus may adversely influence the definition of a probability distribution. If an aircraft has diverged from the fleet structural configuration at the analysis location (ie due to repair), then that aircraft should also be excluded from the analysis.

4.2.4 Data Point

A data point comprises the AFHRS and ranked cumulative probability (if cracked) of an aircraft at the analysis location.

4.3 Use of Cracking versus Failure Data

In many aircraft applications, cracking data will be more common than failure data, particularly for primary structure. This is because cracking data from inspections as part of a DTA safety approach will normally be used to determine cracking and failure probabilities.

Within the template, it is assumed that cracking has a lognormal probability distribution. For relatively small nominal baseline crack lengths, this is analogous to crack initiation.

Depending upon the application, only failure data may be available. In this case, substitute failure data for cracking data in the template and set crack growth AFHRS to zero. Using this approach implicitly assumes that the failure data has a lognormal probability distribution.

4.4 Inclusion Criteria for Aircraft in the Fleet

Only aircraft for which the cracking status at the analysis location is known should be included in the fleet. For instance, if inspection data is only available for 20 out of the 30 aircraft in the operational fleet, then only the inspected 20 aircraft constitute the fleet for the purposes of defining the probability distribution. In this case it is assumed that the inspected aircraft are representative of the fleet, and will not slew the results. The lack of inspection

means that it is unknown if the location is cracked or uncracked, and therefore these aircraft must be left out of the analysis.

Note that in the absence of directed inspections at the analytical location, the absence of cracking records may be sufficient to qualify the aircraft as uncracked if typical maintenance practices would be expected to detect such damage.

4.5 Use of Multiple Inspection Data

Only one data point should be used for each location on each aircraft. Thus, if cracking has been found, then that cracking data point (AFHRS) should be used. If no cracking has been found, then the data point (AFHRS) for the last inspection (or relevant maintenance event) should be used.

Hence, if three aircraft have each been inspected at a particular location three times, and cracking was found on the third aircraft during the second inspection, then:

- the AFHRS for the second inspection of the third aircraft should be used,
- the AFHRS for the third inspection of the first and second aircraft should be used.

4.6 Use of Data from Multiple Structural Locations

Where appropriate, cracking data from several similar structural locations on the aircraft may be collated and treated as a single data set. An example of similar structural locations may be a line of fasteners holes in a panel with similar geometry and loading.

If this approach is used, then the criteria regarding inclusion of aircraft in the fleet (Chapter 4.4) and multiple inspection data (Chapter 4.5) are applied to each individual location. Only after this step can the data points for all locations be collated together. Hence, the exact locations included in the analysis may be different for each aircraft.

Due to the number of similar structural locations on an aircraft (ie panel fastener holes), significant effort is required to identify the configuration and cracking/inspection history of every location on every aircraft, as required to accurately filter the data. However, the economics of scale that are achieved by combining locations permit some approximations to be made. For instance, let us assume that data from 20 similar structural locations (fastener holes) on a fleet of 40 aircraft are being collated for an analysis. If it is known that a couple of locations on a couple of aircraft have been modified, it may be possible to ignore these irregularities with minimal impact upon the analysis (assuming these locations did not crack after the modification was incorporated). This simplification may save significant effort in identifying the exact locations and aircraft that have been modified.

4.7 Use of Cracking Data Corresponding to Various Crack Sizes

As stated in Chapter 4.2.1, it is important that all cracking data pertains to a crack of a nominal fixed length. In reality, cracks will be found at a range of crack sizes. Hence, it may be necessary to normalise the data to a nominal crack size.

If a crack growth curve is available for the location of interest, then the AFHRS for each crack may be normalised using the difference in AFHRS between the actual crack length and the nominal crack length. To minimise the effort required to normalise large data sets, the data may be grouped into subsets with the average crack length of each subset used to adjust the AFHRS of all data points within that subset.

If crack growth data is not known, it is still possible to conduct an analysis. However, the results will be based on a crack of no defined size. When using these results, the largest crack should be considered as the likely size as this will lead to a conservative solution.

4.8 Determination of Cumulative Probability of Failure Distribution

The cumulative probability of failure distribution may be determined directly from the lognormal cumulative probability of cracking distribution by the addition of a crack growth rate distribution. According to Reference 2, there are numerous crack growth rate distributions which may be modelled, but the following two assumptions bound the problem:

1. Assume that all cracks, regardless of the time to reach a nominal length, grow as estimated by a best estimate crack growth curve (ie constant crack growth rate).
2. Assume that all cracks grow with a distribution of rates that is the same as the distributions of time to reach the nominal length.

Assumption 1 is equivalent to adding a constant crack growth rate to the cumulative probability of cracking distribution. The resulting distribution will not produce a straight line and is hence not lognormal. Assumption 2 is equivalent to assuming a single initial crack size to which a distribution of crack growth rates is added. The resulting distribution is lognormal.

The two assumptions are illustrated in Figure 7 (assuming the same mean crack growth rate). As can be seen, the two resulting distributions are the same at a cumulative probability of 0.5. However for cumulative probabilities less than 0.5, the distribution of crack growth rates lead to a more conservative result compared to the constant crack growth rate assumption. The opposite is true for cumulative probabilities above 0.5.

At low cumulative probabilities (ie less than 0.001), the distribution of crack growth rates curve combines the fastest initiating crack with the fastest growing crack. This was considered overly conservative. Consequently, the cumulative probability of failure distribution was determined directly from cumulative probability of cracking distribution by adding the number of crack growth AFHRS from cracking to failure (ie Assumption 1).

4.9 Use of Data from a Fatigue Test Article

A fatigue test article can be treated similar to any other fleet. The only difference is that there is only one data point available at any given location. For a test intended to represent the 50th percentile aircraft, this data point will be located at a cumulative probability equal to 0.5. For a test intended to represent the 85th percentile aircraft, this data point will be located at a cumulative probability equal to 0.15.

If fatigue test data from multiple structural locations are being collated, a single data point can be obtained by averaging the AFHRS.

If sufficient locations have been located, it may be possible to plot each of these points and fit a trendline to the data. In this situation, the relative severity of the fatigue test article compared to the fleet of interest may need to be addressed (refer Chapter 4.10).

4.10 Leveraging Off Other Fleets (Including Fatigue Test Article)

Suppose Fleet "A" contains 200 aircraft with an average of 15,000 AFHRS. Fleet "B" contains 20 structurally similar aircraft with an average of 10,000 AFHRS. Assuming identical usage, it is expected that Fleet "A" will exhibit more cracking per aircraft at any given location, and will have a larger dataset, than Fleet "B". It is therefore desirable for the cumulative probability distribution of Fleet "A" to be applied to Fleet "B".

However, it is unlikely that two such fleets would be operated in the same manner. It is therefore necessary to adjust the lognormal cumulative probability of cracking distribution to reflect this change in usage. One method of achieving this adjustment is a horizontal shift of the distribution to reflect a change in mean AFHRS. It is assumed that the standard deviation, and hence slope of the line defining the lognormal cumulative probability distribution, does not change.

The shift in mean AFHRS from Fleet "A" to Fleet "B" may be calculated using the relative severity of the two fleets at the analysis location (with respect to cracking). Thus:

$$Fleet"B"MeanAFHRS = Fleet"A"MeanAFHRS \times \frac{Fleet"A"Severity}{Fleet"B"Severity}$$

Hence, in the above example, if the Fleet "B" was 1.5 times more severe than Fleet "A", the Fleet "B" mean AFHRS for the cumulative probability of cracking distribution would be 10,000 AFHRS (ie 15,000 AFHRS × 1/1.5).

The relative severity of two fleets at an analysis location (with respect to cracking) may be determined from comparisons of life to a defined crack size ("crack initiation") or crack growth intervals.

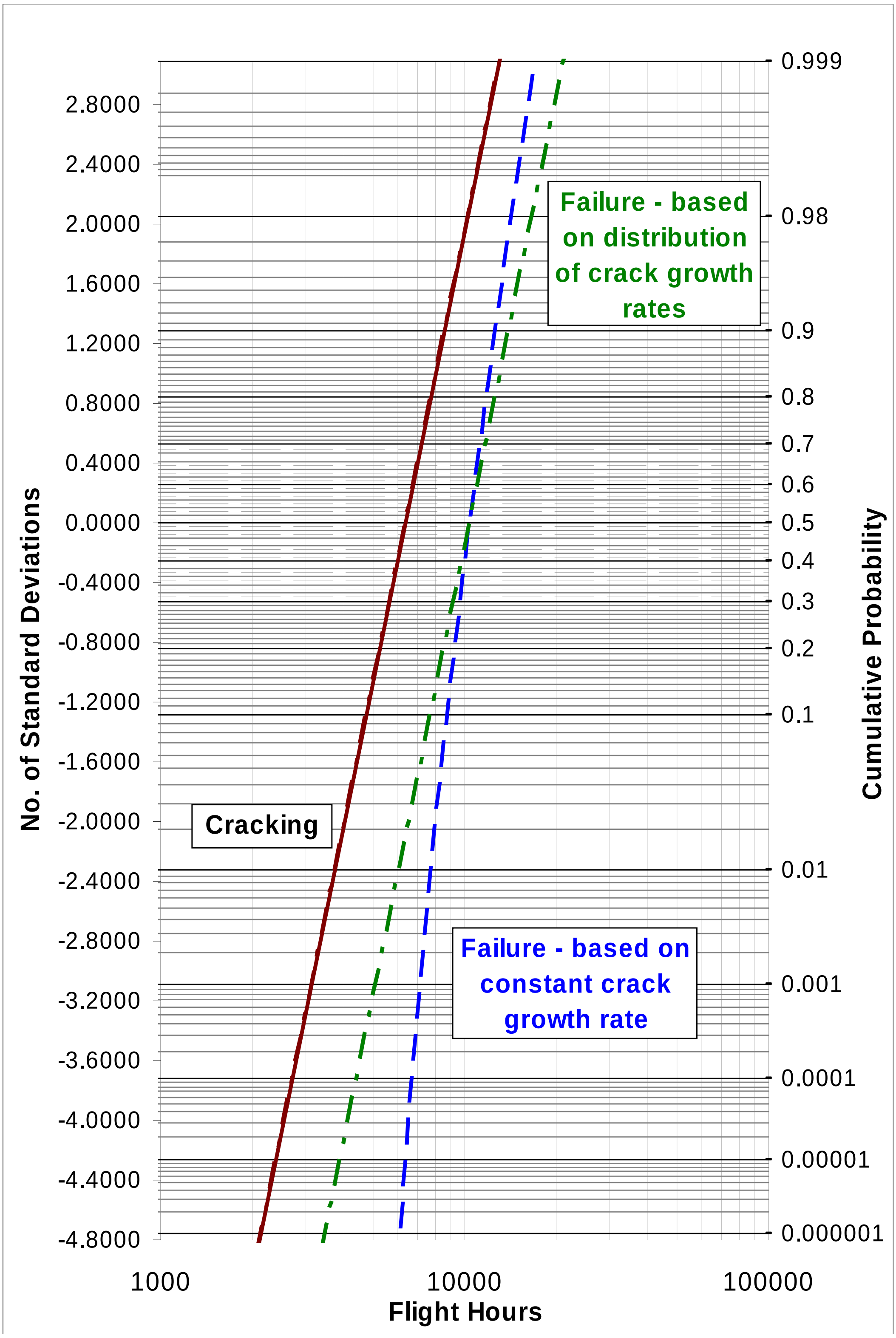

Figure 7: Cumulative Probability of Cracking and Failure

5. The Cumulative Probability Chart Template

5.1 Introduction

A Microsoft Excel workbook was developed as a cumulative probability chart template to automate the generation of cumulative probability charts for aircraft cracking and failure data. This enables accurate and fast analysis to occur, with many locations or scenarios able to be generated.

Excel has several limitations when handling statistical data. However as this program is in widespread use, affords the user transparency of the calculations, and provides ease of modification (presentation and calculation), it was considered the best readily available platform for the template.

5.2 Overview of the Template File

The Excel file that comprises the cumulative probability chart template is named 'Cumulative_Probability_Chart_Template.xls'.

Within this file there are three worksheets, titled:

- 'Prob Chart for Summary Data'
- 'Prob Chart for Detailed Data', and
- 'Template Revision Status'.

The 'Prob Chart for Summary Data' worksheet is a cumulative probability chart template if there is only one data point available or multiple data points have been averaged to obtain one data point. Detailed information regarding this template worksheet is found in Chapter 6.

The 'Prob Chart for Detailed Data' worksheet is a cumulative probability chart template that permits multiple data points to be entered and a trendline to be fitted. Detailed information regarding this template worksheet is found in Chapter 7.

The 'Template Revision Status' worksheet contains basic information regarding the revision status of the template. At the time of writing, the template was Revision 1.

5.3 Assumptions

Assumptions implicit in this workbook are:

- fleet cracking has a lognormal distribution with respect to AFHRS,
- the sample size of the fleet data is statistically representative,
- all cracking data corresponds to a nominal crack length (the value of the nominal crack length is not important),
- the Fleet 'A' and Fleet 'B' lognormal cumulative probability of cracking distributions have identical standard deviations,

- the ratio between the means of the Fleet 'A' and Fleet 'B' lognormal cumulative probability of cracking distributions can be wholly defined by the relative severities of these two fleets at the analysis location (with respect to cracking), and
- the distribution of failure in Fleet 'B' can be estimated using the lognormal cumulative probability of cracking distribution and the number of AFHRS required to grow the crack from the nominal crack length to the critical crack length.

6. The Cumulative Probability Worksheet for Summary Data

6.1 Introduction

This Chapter provides a guide to the 'Prob Chart for Summary Data' worksheet.

Information is provided regarding:

- the purpose of the worksheet
- the layout of the worksheet
- how to enter data
- the output data presented

While basic information and guidance regarding the use of input data is provided in this Chapter, it is assumed that the reader is familiar with the content and guidance presented in Chapters 3 and 4.

6.2 Purpose of the Worksheet

The purpose of the worksheet is as follows:

- Calculate a cumulative probability distribution for cracking in Fleet 'A' based on a single data point and an assumed standard deviation.
- Calculate a cumulative probability distribution for cracking in Fleet 'B' based upon the cumulative probability distribution for cracking in Fleet 'A' and the relative usage severity of Fleet 'A' and Fleet 'B'.
- Calculate the cumulative probability distribution for failure in Fleet 'B' based upon the cumulative probability distribution for cracking in Fleet 'B' and the assumed crack growth interval between cracking and failure (in Fleet 'B').
- Calculate the AFHRS corresponding to a user specified cumulative probability of cracking or failure (user option).
- Calculate a cumulative probability of cracking or failure (user option) at a user specified AFHRS.
- Calculate the Fatigue Index (FI) at which inspections should begin, based upon the AFHRS corresponding to the user specified cumulative probability of cracking or failure (user option) and the fleet average FI rate (per 1,000 AFHRS).

- Present the cumulative probability distributions and associated data on a cumulative probability chart.

6.3 Layout of the Worksheet

The worksheet contains several windows which are defined by yellow coloured cells with black borders (as well as the Excel chart window). The primary windows are:

- the input/output data window (cells B2 to E47)
- the chart window (cells G1 to G48)
- the cumulative probability calculation windows (cells I1 to N71)
- the cumulative probability chart gridlines window (cells Q1 to T290)

An example input/output data window and chart window are presented in Figure 8. The corresponding cumulative probability calculation windows are presented in Figure 9. A section of the cumulative probability chart gridlines window is presented in Figure 10.

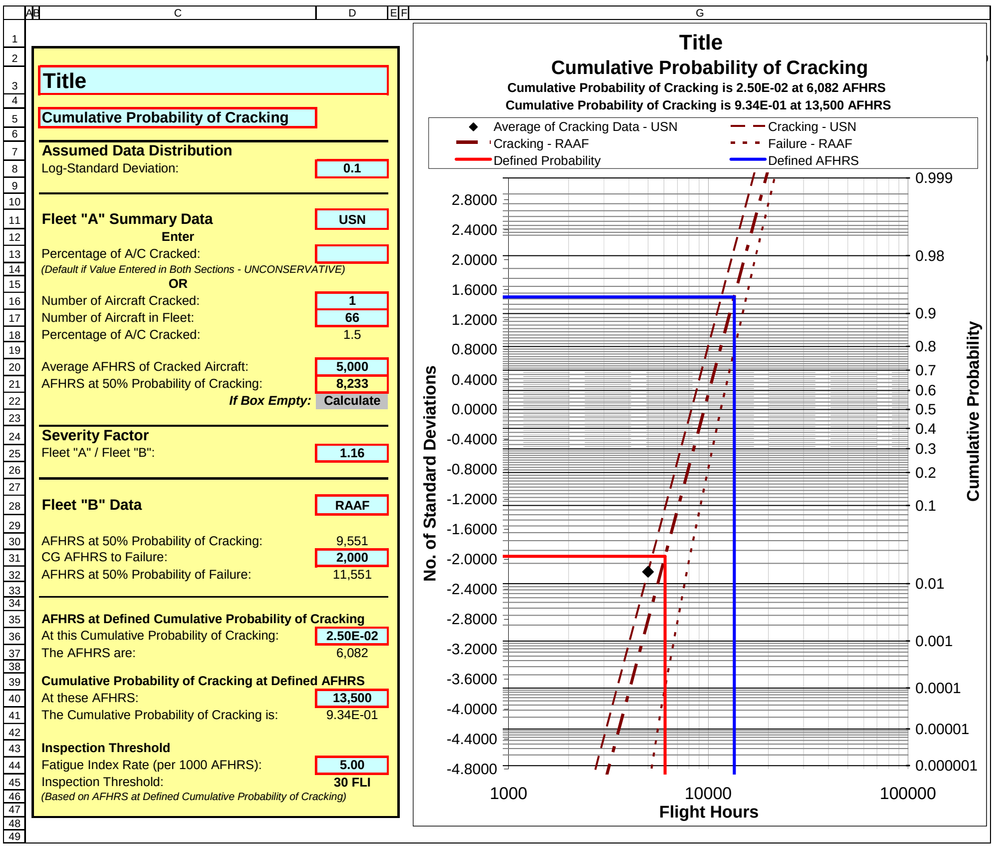

Figure 8: Example Input/Output Data Window and Chart Window

	I	J	K	L	M	N
1	**Probability Data**	**Initial Value**	**Fleet "A"**	**Fleet "B"**	**Fleet "B" + Crack Growth**	
2	**Mean**	5,000	8233.46	9550.81	11550.81	
3	**LOG(Mean)**		3.9156	3.9800	4.0626	
4	**logSD**		0.1	0.1		
5	**Average AFHRS**		5,000			
6	**Average Cumulative Probability of Average Cracked**		1.5	**-2.166123505** No. of SD for % of Cracking		
7	**Fleet "A" / Fleet "B" Severity Factor**		1.16			
8	**Fleet "B" CG AFHRS to Failure**		2000.00			
9	**Calculated Probability of Cracking**		0.015150875			
10	**Difference**		-6.39891E-07			
11	**Allowable Difference**		1.00E-04			
12						
13	**Fleet "A" Line**					
14	**AFHRS**	**LOG(AFHRS)**		**cdf**	**No. of SD**	
15	1	0		0	-39.15582355	
16	100000	5		1	10.84417645	
17						
18	**Fleet "B" Line**					
19	**AFHRS**	**LOG(AFHRS)**		**cdf**	**No. of SD**	
20	1	0		0	-39.80040344	
21	100000	5		1	10.19959656	
22						
23	**Specific Points on the Fleet "B" Probability of Cracking or Failure Curve**				**Cum. Prob.**	**AFHRS**
24	**Defined Probability of Crack Being Present or Failure**				2.50E-02	6082
25	**Defined Flight Hours**				9.34E-01	13500
26	**Cumulative Probability of Cracking**		**Cumulative Probability of Cracking is 2.50E-02 at 6,082 AFHRS**			
27	**Cumulative Probability of Failure**		**Cumulative Probability of Cracking is 9.34E-01 at 13,500 AFHRS**			
28						
29	**Desired Probability Intercept Lines**					
30	**AFHRS**		**LOG(AFHRS)-Cracking**		**Defined cdf**	**No. of SD**
31	1		3.784E+00		2.500E-02	-1.960E+00
32	6,082		3.784E+00		2.500E-02	-1.960E+00
33	6,082		3.784E+00		2.500E-02	-1.960E+00
34	6,082		3.460E+00		1.000E-07	-5.199E+00
35	**Desired AFHRS Intercept Lines**					
36	**Defined AFHRS**		**LOG(AFHRS)-Cracking**		**cdf**	**No. of SD**
37	1		4.130E+00		9.336E-01	1.503E+00
38	13,500		4.130E+00		9.336E-01	1.503E+00
39	13,500		4.130E+00		9.336E-01	1.503E+00
40	13,500		3.460E+00		1.000E-07	-5.199E+00
41						
42	**Fleet "B" Cracking and Failure Lines**					
43	**AFHRS - Cracking**	**AFHRS - Failure**	**LOG(AFHRS)**		**cdf**	**No. of SD**
44	9.550813047	2009.55	0.980040344		4.907E-198	-3.000E+01
45	15.13701858	2015.14	1.180040344		8.124E-173	-2.800E+01
46	23.9905577	2023.99	1.380040344		2.476E-149	-2.600E+01
47	38.02247159	2038.02	1.580040344		1.390E-127	-2.400E+01
48	60.26155638	2060.26	1.780040344		1.440E-107	-2.200E+01
49	95.50813047	2095.51	1.980040344		2.754E-89	-2.000E+01
50	151.3701858	2151.37	2.180040344		9.741E-73	-1.800E+01
51	239.905577	2239.91	2.380040344		6.389E-58	-1.600E+01
52	380.2247159	2380.22	2.580040344		7.794E-45	-1.400E+01
53	602.6155638	2602.62	2.780040344		1.776E-33	-1.200E+01
54	955.0813047	2955.08	2.980040344		7.620E-24	-1.000E+01
55	1513.701858	3513.70	3.180040344		6.221E-16	-8.000E+00
56	2399.05577	4399.06	3.380040344		9.866E-10	-6.000E+00
57	3802.247159	5802.25	3.580040344		3.167E-05	-4.000E+00
58	6026.155638	8026.16	3.780040344		2.275E-02	-2.000E+00
59	9550.813047	11550.81	3.980040344		5.000E-01	0.000E+00
60	15137.01858	17137.02	4.180040344		9.772E-01	2.000E+00
61	23990.5577	25990.56	4.380040344		1.000E+00	4.000E+00
62	38022.47159	40022.47	4.580040344		1.000E+00	6.000E+00
63	60261.55638	62261.56	4.780040344		1.000E+00	8.000E+00
64	95508.13047	97508.13	4.980040344		1.000E+00	1.000E+01
65						
66	**Chart Series Titles**					
67	Average of Cracking Data - USN					
68	Cracking - USN					
69	Cracking - RAAF					
70	Failure - RAAF					
71						

Figure 9: Example Cumulative Probability Calculation Windows

	P	Q	R	S	T	U
1		**Gridlines for Cumulative Probability Chart**				
2		**Minimum AFHRS**		**1**		
3		**Maximum AFHRS**		**100000**		
4		**Index**	**cdf**	**AFHRS**	**No. of SD**	
5		1	0.999	1	3.0902	
6		1	0.999	100000	3.0902	
7		2	0.998	1	2.8782	
8		2	0.998	100000	2.8782	
9		3	0.997	1	2.7478	
10		3	0.997	100000	2.7478	
11		4	0.996	1	2.6521	
12		4	0.996	100000	2.6521	
13		5	0.995	1	2.5758	
14		5	0.995	100000	2.5758	
15		6	0.994	1	2.5121	
16		6	0.994	100000	2.5121	
17		7	0.993	1	2.4573	
18		7	0.993	100000	2.4573	
19		8	0.992	1	2.4089	
20		8	0.992	100000	2.4089	
21		9	0.991	1	2.3656	
22		9	0.991	100000	2.3656	
23		10	0.99	1	2.3263	
24		10	0.99	100000	2.3263	
25		11	0.98	1	2.0537	
26		11	0.98	100000	2.0537	
27		12	0.97	1	1.8808	
28		12	0.97	100000	1.8808	
29		13	0.96	1	1.7507	
30		13	0.96	100000	1.7507	
31		14	0.95	1	1.6449	
32		14	0.95	100000	1.6449	
33		15	0.94	1	1.5548	
34		15	0.94	100000	1.5548	
35		16	0.93	1	1.4758	
36		16	0.93	100000	1.4758	
37		17	0.92	1	1.4051	
38		17	0.92	100000	1.4051	
39		18	0.91	1	1.3408	
40		18	0.91	100000	1.3408	
41		19	0.9	1	1.2816	
42		19	0.9	100000	1.2816	
43		20	0.89	1	1.2265	
44		20	0.89	100000	1.2265	

Figure 10: Example Cumulative Probability Chart Gridlines Window

6.4 How to Enter Data

This section presents a step by step guide to entering information into the worksheet.

Step	Description / *Guidance*	Cell Ref.
1.	**Enter Title** *Enter a title for the cumulative probability data and chart. It is recommended that this title provide useful information regarding aircraft type and/or structural location.*	**C3**
2.	**Choose Between Probability of Cracking or Probability of Failure** *Click on the cell and from the pull down list select either "Probability of Cracking" or "Probability of Failure".* *If "Probability of Cracking" is selected, then the AFHRS corresponding to a user specified cumulative probability of cracking is calculated. Likewise the cumulative probability of cracking at a user specified AFHRS is calculated.* *If "Probability of Failure" is selected, then the AFHRS corresponding to a user specified cumulative probability of failure is calculated. Likewise the cumulative probability of failure at a user specified AFHRS is calculated.*	**C5**
3.	**Enter an Assumed Standard Deviation for the Lognormal Cumulative Probability of Cracking Distribution** *This assumed standard deviation may be based on historical data, the standard deviation for similar locations, or other sources.*	**D8**
4.	**Enter a Name for Fleet "A" (Optional)** *If desired, enter a name for Fleet 'A'. Ideally, use short names or acronyms to avoid errors in the presentation of the data (due to overlapping or wrapping of data text). If no name is entered, the fleet will be referred to as Fleet 'A'.*	**D11**
5.	**Enter the Percentage of Aircraft Cracked** **(Not Required if Data Entered in Step 6)** *Enter the percentage of aircraft in Fleet 'A' that are cracked. This information is not required if data regarding the fleet size and number of aircraft cracked will be entered in Step 6.* *However, if data is entered in both steps, the percentage of aircraft cracked entered in this step will be used in the calculations.* ***Warning – Using this type of input will result in unconservative cumulative probability distributions, particularly for small fleet sizes (Refer to Chapter 3 for further information).***	**D13**
6.	**Enter the Number of Aircraft Cracked and the Number of Aircraft in the Fleet.** **(Not Required if Data Entered in Step 5)** *Enter the number of aircraft cracked in Fleet 'A' and the number of aircraft in the Fleet 'A'.* *This information is not required if the percentage of fleet cracked was entered in Step 5. However, if data is entered in both steps, the percentage of aircraft cracked entered in Step 5 will be used in the calculations.* *If data for multiple locations is being used, ensure that the entered data reflects the number of locations rather than the number of aircraft (Refer Chapter 4.6).*	**D16 & D17**

Step	Description / *Guidance*	Cell Ref.
7.	**Enter the Average AFHRS of the Cracked Aircraft.** *Enter the average AFHRS of the cracked aircraft in Fleet 'A'.* *Refer to Chapter 3 for guidance regarding the averaging of data points.*	**D20**
8.	**Click on the 'Calculate' Command Button.** *Click on the 'Calculate' command button (cell D22) to run a macro that calculates the mean AFHRS for Fleet 'A' based upon the assumed standard deviation. The mean AFHRS is the AFHRS corresponding to a cumulative probability of 0.5 (50%).* *This macro is named 'Solve_for_Mean_AFHRS' and can also be run using the 'Ctrl + m' shortcut key.* *This macro utilises the Excel Solver add-in. Therefore, if no solution can be found (indicated by "#NUM!" in cell D21), then the initial starting value for the mean AFHRS may need to be adjusted. The initial starting value for the mean AFHRS calculation is stored in cell J2. Adjust this initial value closer to an estimate of the mean AFHRS and click on the 'Calculate' command button to rerun the macro.*	**D22**
9.	**Enter a Severity Factor.** *Enter the severity factor that relates the usage severity of Fleet 'A' and Fleet 'B'. This severity factor is used to calculate the mean AFHRS for Fleet 'B' from the mean AFHRS of Fleet 'A'. The severity factor is equal to the usage severity of Fleet 'A' divided by the usage severity of Fleet 'B'.* *If no severity factor is entered, a severity factor of 1 is assumed (ie Fleet 'A' and Fleet 'B' have the same usage severity). This option is useful if the cumulative probability of failure of Fleet 'A' is required (it will be the same as Fleet 'B' when the severity factor is equal to 1).* *Further information regarding calculation and use of the severity factor is presented in Chapter 4.10.*	**D25**
10.	**Enter a Name for Fleet "B" (Optional)** *If desired, enter a name for Fleet 'B'. Ideally, use short names or acronyms to avoid errors in the presentation of the data (due to overlapping or wrapping of data text). If no name is entered, the fleet will be referred to as Fleet 'B'.*	**D28**
11.	**Enter the Number of AFHRS for Crack Growth from Cracking to Failure. (Optional)** *Enter the number of AFHRS required for cracking to progress to failure under Fleet 'B' usage.* *These AFHRS may be calculated using a crack growth curve for the location under consideration, with failure assumed to occur at the critical crack size (based on residual strength). The initial crack size is equal to the crack size on which the cracking data was derived.* *Refer to Chapter 4 for further guidance regarding determination of the initial crack size.*	**D31**
12.	**Enter the Cumulative Probability of Cracking / Failure for which the Corresponding AFHRS are Required. (Optional)** *If "Probability of Cracking" was selected in Step 2, then enter the cumulative probability of cracking for which the corresponding AFHRS are required. For example, enter "0.001" to calculate the AFHRS at which the cumulative probability of cracking is equal to 1 in 1000.* *If "Probability of Failure" was selected in Step 2, then the AFHRS at which the cumulative probability of failure is equal to cell D36 will be calculated. These calculations are based on the Fleet 'B' cumulative probability of crack / failure.*	**D36**

Step	Description / *Guidance*	Cell Ref.
13.	**Enter the AFHRS for which the Cumulative Probability of Cracking / Failure is Required. (Optional)** *If "Probability of Cracking" was selected in Step 2, then enter the AFHRS at which the cumulative probability of cracking is required. For example, enter "10,000" to calculate the cumulative probability of cracking at 10,000 AFHRS.* *If "Probability of Failure" was selected in Step 2, then the cumulative probability of failure will be calculated for the entered AFHRS. These calculations are based on the Fleet 'B' cumulative probability of crack / failure.*	**D40**
14.	**Enter the Fatigue Index (FI) Rate for Fleet 'B' (Optional).** *Enter the average Fatigue Index (FI) accrual rate for Fleet 'B' in terms of FI per 1,000 AFHRS.* *This number is used to calculate an approximate FI for the inspection threshold, assuming inspections are to begin at the AFHRS calculated in Step 12 (at the user specified cumulative probability of cracking).*	**D44**
15.	**Revise Entries as Required (Optional).** *Data entered in previous steps may be revised at any time.* *If one or more of the following entries are revised, the mean AFHRS of Fleet 'A' will need to be manually recalculated (performing Step 8 again):* *a. assumed standard deviation (Step 3),* *b. percentage of aircraft cracked (Step 5),* *c. number of aircraft cracked (Step 6),* *d. number of aircraft in the fleet (Step 6), or* *e. average AFHRS of the cracked aircraft (Step 7).* *Revision of all other entries will be automatically incorporated.*	**See Steps 1 to 14**

6.5 Output Data Presented

All relevant output data is presented in the input/output data window and chart window (refer to Figure 8). In addition to the input data, the input/output data window contains the output data presented in Table 1.

Table 1: Output Data Presented in Input/Output Window

Description of Output Data	Cell Location
Percentage of aircraft cracked (if the number of aircraft cracked and number of aircraft in the fleet were entered in step 6).	D18
AFHRS corresponding to a cumulative probability of cracking in Fleet 'A' of 0.5 (50%).	D21
AFHRS corresponding to a cumulative probability of cracking in Fleet 'B' of 0.5 (50%).	D30
AFHRS corresponding to a cumulative probability of failure in Fleet 'B' of 0.5 (50%).	D32
AFHRS corresponding to the user defined cumulative probability of cracking / failure (user option in cell C5).	D37
Cumulative probability of cracking / failure corresponding to the user defined AFHRS (user option in cell C5).	D41
Fatigue Index (FI) for the inspection threshold, assuming inspections are to begin at the AFHRS corresponding to the user defined cumulative probability of cracking (cell D36).	D45

The chart window presents the information presented in Table 2.

Table 2: Output Data Presented in Chart Window

Output Data	Description
Title (As entered in cell C3)	Chart Heading
Probability of cracking or probability of failure (as selected by the user in cell C5)	Chart Sub-heading
User defined cumulative probability of cracking (or cumulative probability of failure) and corresponding AFHRS	Text at Top of Chart under Headings
User defined AFHRS and corresponding cumulative probability of cracking (or cumulative probability of failure)	Text at Top of Chart under Headings
Data Point Representing Average AFHRS of Fleet 'A' Cracked Aircraft	Black Filled Diamond
Cumulative probability of cracking distribution for Fleet 'A'.	Large Brown Dashed Line
Cumulative probability of cracking distribution for Fleet 'B'.	Large Brown Dashed-Dotted Line
Cumulative probability of failure distribution for Fleet 'B'.	Small Brown Dashed Line
AFHRS corresponding to the user defined cumulative probability of cracking / failure.	Red Solid Line
Cumulative probability of cracking / failure corresponding to the user defined AFHRS.	Blue Solid Line

The calculations and macro operations that form the basis of this template are described in Appendix A.

7. The Cumulative Probability Worksheet for Detailed Data

7.1 Introduction

This Chapter provides a guide to the 'Prob Chart for Fleet Data' worksheet.

Information is provided regarding:

- the purpose of the worksheet
- the layout of the worksheet
- how to enter data
- the output data presented

While basic information and guidance regarding the use of input data is provided in this Chapter, it is assumed that the reader is familiar with the content and guidance presented in Chapters 3 and 4.

7.2 Purpose of the Worksheet

The purpose of the worksheet is as follows:

- Plot the ranked cumulative probabilities for multiple data points in Fleet 'A' and identify any outliers.
- Fit a cumulative probability distribution (trendline) to the Fleet 'A' data points (excluding outliers) and define the mean AFHRS and standard deviation of the Fleet 'A' fitted trendline.
- Calculate a cumulative probability distribution for cracking in Fleet 'A' based on the mean AFHRS of the Fleet 'A' fitted trendline and an assumed standard deviation.
- Calculate a cumulative probability distribution for cracking in Fleet 'B' based upon the cumulative probability distribution for cracking in Fleet 'A' and the relative usage severity of Fleet 'A' and Fleet 'B'.
- Calculate the cumulative probability distribution for failure in Fleet 'B' based upon the cumulative probability distribution for cracking in Fleet 'B' and the assumed crack growth interval between cracking and failure (in Fleet 'B').
- Calculate the AFHRS corresponding to a user specified cumulative probability of cracking or failure (user option).
- Calculate a cumulative probability of cracking or failure (user option) at a user specified AFHRS.
- Calculate the Fatigue Index (FI) at which inspections should begin, based upon the AFHRS corresponding to the user specified cumulative probability of cracking or failure (user option) and the fleet average FI rate (per 1,000 AFHRS).
- Present the cumulative probability distributions and associated data on a cumulative probability chart.

7.3 Layout of the Worksheet

The worksheet contains several windows which are defined by yellow coloured cells with black borders (as well as the Excel chart window). The primary windows are:

- the input/output data window (cells B2 to E47)
- the chart window (cells G1 to G47)
- the cumulative probability calculation windows (cells I1 to N73)
- the fleet data and fitted trendline windows (cells Q1 to AA10000)
- the cumulative probability chart gridlines window (cells AC1 to AF292)

An example input/output data window and chart window are presented in Figure 11. The corresponding cumulative probability calculation windows are presented in Figure 12. The fleet data and fitted trendline windows are presented in Figure 13. The cumulative probability chart gridlines window is presented in Figure 14.

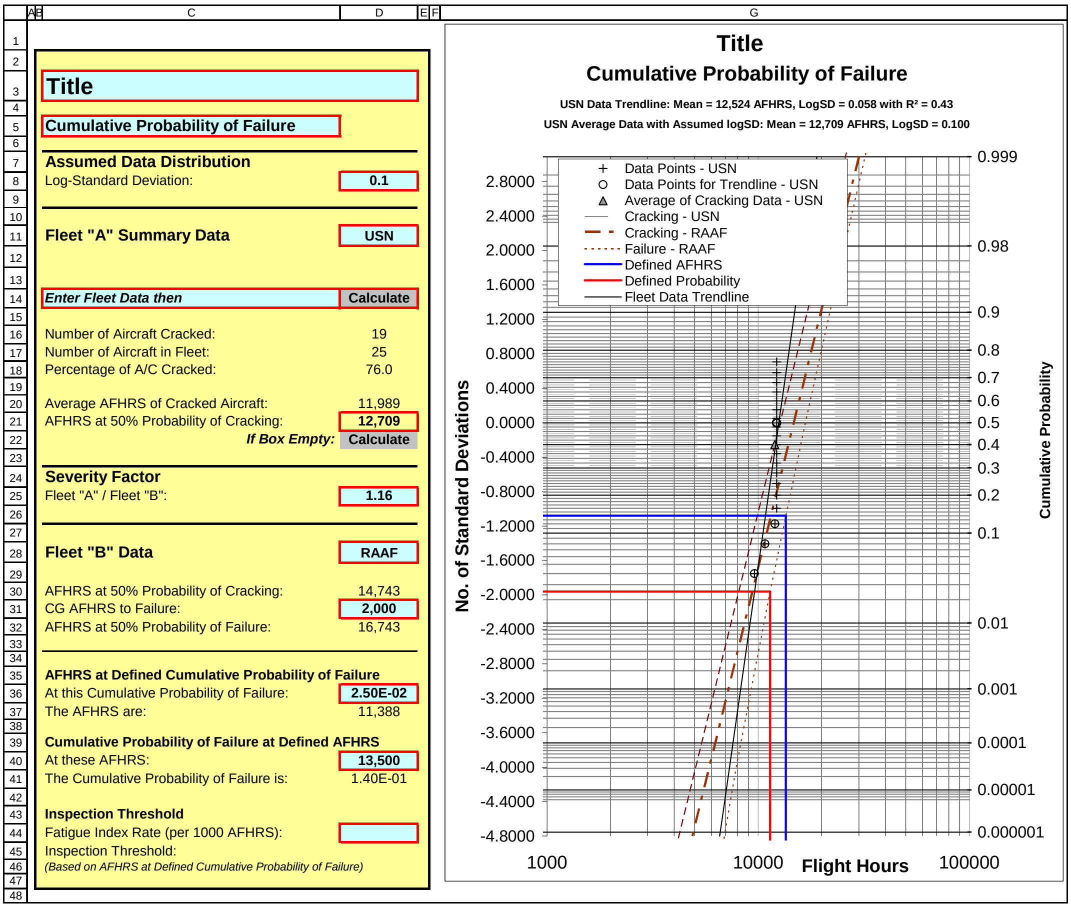

Figure 11: Example Input/Output Data Window and Chart Window

	I	J	K	L	M	N
1	**Probability Data**	**Initial Value**	**Fleet "A"**	**Fleet "B"**	**Fleet "B" + Crack Growth**	
2	**Mean (AFHRS)**	12,013	12709.34	14742.84	16742.84	
3	**Log(Mean)**		4.1041	4.1686	4.2238	
4	**logSD**		0.1	0.1		
5	**Average AFHRS**		11,989			
6	**Average Cumulative Probability of Average Cracked**		40.0	**-0.253348013**	No. of SD for % of Cracking	
7	**Fleet "A" / Fleet "B" Severity Factor**		1.16			
8	**Fleet "B" CG AFHRS to Failure**		2000.00			
9	**Calculated Probability of Cracking**		0.399999648			
10	**Difference**		-3.51694E-07			
11	**Allowable Difference**		1.00E-04			
12						
13	**Fleet "A" Line**					
14	**AFHRS**	**LOG(AFHRS)**		**cdf**	**No. of SD**	
15	1	0		0	-41.04123035	
16	100000	5		1	8.95876965	
17						
18	**Fleet "B" Line**					
19	**AFHRS**	**LOG(AFHRS)**		**cdf**	**No. of SD**	
20	1	0		0	-41.68581024	
21	100000	5		1	8.314189758	
22						
23	**Specific Points on the Fleet "B" Probability of Cracking or Failure Curve**				**Cum. Prob.**	**AFHRS**
24	**Defined Probability of Crack Being Present or Failure**				2.50E-02	11388
25	**Defined Flight Hours**				1.40E-01	13500
26	**Cumulative Probability of Cracking**		**Cumulative Probability of Failure is 2.50E-02 at 11,388 AFHRS**			
27	**Cumulative Probability of Failure**		**Cumulative Probability of Failure is 1.40E-01 at 13,500 AFHRS**			
28						
29	**Desired Probability Intercept Lines**					
30	**AFHRS**		**LOG(AFHRS)-Cracking**		**Defined cdf**	**No. of SD**
31	1		3.973E+00		2.500E-02	-1.960E+00
32	11,388		3.973E+00		2.500E-02	-1.960E+00
33	11,388		3.973E+00		2.500E-02	-1.960E+00
34	11,388		3.649E+00		1.000E-07	-5.199E+00
35	**Desired AFHRS Intercept Lines**					
36	**Defined AFHRS**		**LOG(AFHRS)-Cracking**		**cdf**	**No. of SD**
37	1		4.061E+00		1.403E-01	-1.079E+00
38	13,500		4.061E+00		1.403E-01	-1.079E+00
39	13,500		4.061E+00		1.403E-01	-1.079E+00
40	13,500		3.649E+00		1.000E-07	-5.199E+00
41						
42	**Fleet "B" Cracking and Failure Lines**					
43	**AFHRS - Cracking**	**AFHRS - Failure**	**LOG(AFHRS)**		**cdf**	**No. of SD**
44	14.74283565	2014.74	1.168581024		4.907E-198	-3.000E+01
45	23.36581986	2023.37	1.368581024		8.124E-173	-2.800E+01
46	37.03232884	2037.03	1.568581024		2.476E-149	-2.600E+01
47	58.69228587	2058.69	1.768581024		1.390E-127	-2.400E+01
48	93.02100433	2093.02	1.968581024		1.440E-107	-2.200E+01
49	147.4283565	2147.43	2.168581024		2.754E-89	-2.000E+01
50	233.6581986	2233.66	2.368581024		9.741E-73	-1.800E+01
51	370.3232884	2370.32	2.568581024		6.389E-58	-1.600E+01
52	586.9228587	2586.92	2.768581024		7.794E-45	-1.400E+01
53	930.2100433	2930.21	2.968581024		1.776E-33	-1.200E+01
54	1474.283565	3474.28	3.168581024		7.620E-24	-1.000E+01
55	2336.581986	4336.58	3.368581024		6.221E-16	-8.000E+00
56	3703.232884	5703.23	3.568581024		9.866E-10	-6.000E+00
57	5869.228587	7869.23	3.768581024		3.167E-05	-4.000E+00
58	9302.100433	11302.10	3.968581024		2.275E-02	-2.000E+00
59	14742.83565	16742.84	4.168581024		5.000E-01	0.000E+00
60	23365.81986	25365.82	4.368581024		9.772E-01	2.000E+00
61	37032.32884	39032.33	4.568581024		1.000E+00	4.000E+00
62	58692.28587	60692.29	4.768581024		1.000E+00	6.000E+00
63	93021.00433	95021.00	4.968581024		1.000E+00	8.000E+00
64	147428.3565	149428.36	5.168581024		1.000E+00	1.000E+01
65						
66	**Chart Series Titles**					
67	Average of Cracking Data - USN					
68	Cracking - USN					
69	Cracking - RAAF					
70	Failure - RAAF					
71	Data Points - USN					
72	Data Points for Trendline - USN					
73						

Figure 12: Example Cumulative Probability Calculation Windows

	Q	R	S	T	U	V	W	X	Y	Z	AA
1	**Number of Inspected / Cracked Aircraft**		25	/	19						
2	**LOG Average of Fleet 'A' AFHRS**		12048.19255								
3	**LOG Average of Fleet 'A' Cracked AFHRS**		11989.14558								
4	**Trendline Equation**					**USN Data Trendline: Mean = 12,336 AFHRS, LogSD = 0.054 with R^2 = 0.79**					
5	**Gradient (log(AFHRS)/logSD)**		18.40999273			**USN Average Data with Assumed logSD: Mean = 12,709 AFHRS, LogSD = 0.100**					
6	**log SD (1/Gradient)**		0.054318327								
7	**Intercept**		-75.31839565								
8	**Log (Mean)**		4.091169223								
9	**Mean (AFHRS)**		12335.85405								
10	**R-square**		0.790001136								
11											
12	**Enter Fleet Data**	**Set Cracked=1**						**Trendline Data**		**Data Points for Plotting**	
13	**Unknown AFHRS=Blank**			**Processed Fleet Data**		**Ranked**		**Delete Cells To Discard from**		**DO NOT Delete these Cells**	
14	**Aircraft ID**	**AFHRS**	**Cracked?**	**LOG(AFHRS)**	**AFHRS**	**CDF**	**No. of SD**	**AFHRS**	**No. of SD**	**AFHRS**	**No. of SD**
15	161413	9609	1	3.982678193	9609	0.0400	-1.750686071	9609	-1.7507	9609	-1.7507
16	161765	10772	1	4.032296345	10772	0.0800	-1.40507156	10772	-1.4051	10772	-1.4051
17	160767	12000	1	4.079181246	12000	0.1200	-1.174986792	12000	-1.1750	12000	-1.1750
18	158209	12237	1	4.08767496	12237	0.1600	-0.994457883	12237	0.0000	12237	-0.9945
19	160287	12237	1	4.08767496	12237	0.2000	-0.841621234	12237	0.0000	12237	-0.8416
20	148889	12237	1	4.08767496	12237	0.2400	-0.706302563	12237	0.0000	12237	-0.7063
21	161336	12237	1	4.08767496	12237	0.2800	-0.582841507	12237	0.0000	12237	-0.5828
22	152150	12237	1	4.08767496	12237	0.3200	-0.467698799	12237	0.0000	12237	-0.4677
23	157331	12237	1	4.08767496	12237	0.3600	-0.358458793	12237	0.0000	12237	-0.3585
24	158925	12237	1	4.08767496	12237	0.4000	-0.253347103	12237	0.0000	12237	-0.2533
25	154587	12237	1	4.08767496	12237	0.4400	-0.150969215	12237	0.0000	12237	-0.1510
26	150496	12237	1	4.08767496	12237	0.4800	-0.050153583	12237	0.0000	12237	-0.0502
27	158224	12237	1	4.08767496	12237	0.5200	0.050153583	12237	0.0000	12237	0.0502
28	158934	12237.1	1	4.087678509	12237	0.5600	0.150969215	12237	0.0000	12237	0.1510
29	158567	12237.1	1	4.087678509	12237	0.6000	0.253347103	12237	0.0000	12237	0.2533
30	159884	12237.1	1	4.087678509	12237	0.6400	0.358458793	12237	0.0000	12237	0.3585
31	160285	12237.1	1	4.087678509	12237	0.6800	0.467698799	12237	0.0000	12237	0.4677
32	162771	12237.1	1	4.087678509	12237	0.7200	0.582841507	12237	0.0000	12237	0.5828
33	162777	12237.1	1	4.087678509	12237	0.7600	0.706302563	12237	0.0000	12237	0.7063
34	162772	12237.1									
35	160612	12237.1									
36	161406	12237.1									
37	160766	12237.1									
38	160999	12237.1									
39	157326	12237.1									

Figure 13: Example Fleet Data and Fitted Trendline Windows

	AB	AC	AD	AE	AF	AG
1		**Gridlines for Cumulative Probability Chart**				
2		**Minimum AFHRS**		**1**		
3		**Maximum AFHRS**		**100000**		
4		**Index**	**cdf**	**AFHRS**	**No. of SD**	
5		1	0.999	1	3.0902	
6		1	0.999	100000	3.0902	
7		2	0.998	1	2.8782	
8		2	0.998	100000	2.8782	
9		3	0.997	1	2.7478	
10		3	0.997	100000	2.7478	
11		4	0.996	1	2.6521	
12		4	0.996	100000	2.6521	
13		5	0.995	1	2.5758	
14		5	0.995	100000	2.5758	
15		6	0.994	1	2.5121	
16		6	0.994	100000	2.5121	
17		7	0.993	1	2.4573	
18		7	0.993	100000	2.4573	
19		8	0.992	1	2.4089	
20		8	0.992	100000	2.4089	
21		9	0.991	1	2.3656	
22		9	0.991	100000	2.3656	
23		10	0.99	1	2.3263	
24		10	0.99	100000	2.3263	
25		11	0.98	1	2.0537	
26		11	0.98	100000	2.0537	
27		12	0.97	1	1.8808	
28		12	0.97	100000	1.8808	
29		13	0.96	1	1.7507	
30		13	0.96	100000	1.7507	
31		14	0.95	1	1.6449	
32		14	0.95	100000	1.6449	
33		15	0.94	1	1.5548	
34		15	0.94	100000	1.5548	
35		16	0.93	1	1.4758	
36		16	0.93	100000	1.4758	
37		17	0.92	1	1.4051	
38		17	0.92	100000	1.4051	
39		18	0.91	1	1.3408	
40		18	0.91	100000	1.3408	
41		19	0.9	1	1.2816	
42		19	0.9	100000	1.2816	
43		20	0.89	1	1.2265	
44		20	0.89	100000	1.2265	

Figure 14: Example Cumulative Probability Chart Gridlines Window

7.4 How to Enter Data

This section presents a step by step guide to entering information into the worksheet.

Step	Description / *Guidance*	Cell Ref.
1.	**Enter Title** *Enter a title for the cumulative probability data and chart. It is recommended that this title provide useful information regarding aircraft type and/or structural location.*	**C3**
2.	**Choose Between Probability of Cracking or Probability of Failure** *Click on the cell and from the pull down list select either "Probability of Cracking" or "Probability of Failure".* *If "Probability of Cracking" is selected, then the AFHRS corresponding to a user specified cumulative probability of cracking is calculated. Likewise the cumulative probability of cracking at a user specified AFHRS is calculated.* *If "Probability of Failure" is selected, then the AFHRS corresponding to a user specified cumulative probability of failure is calculated. Likewise the cumulative probability of failure at a user specified AFHRS is calculated.*	**C5**
3.	**Enter a Name for Fleet "A" (Optional)** *If desired, enter a name for Fleet 'A'. Ideally, use short names or acronyms to avoid errors in the presentation of the data (due to overlapping or wrapping of data text). If no name is entered, the fleet will be referred to as Fleet 'A'.*	**D11**
4.	**Enter Data Point Information (Aircraft ID, AFHRS and Whether Aircraft is Cracked) for Fleet 'A'.** *For each aircraft in Fleet 'A', enter the aircraft ID (Column Q) and aircraft AFHRS (Column R) and flag whether or not it is cracked at the location of interest (Column S).* *An aircraft is flagged as cracked at the location of interest if there is an entry in Column S. It does not matter what value is used to flag whether an aircraft is cracked (eg 1, Y), as long as it is used for all cracked aircraft. If different values are used, then the cracked aircraft will not be correctly sorted by AFHRS and calculation of ranked cumulative probabilities for each point will be incorrect.* *If failure data is being used, then indicate whether an aircraft has failed at the location of interest via an entry in Column S (same as for indicating whether an aircraft is cracked).* *If the AFHRS for a particular aircraft is not known, then Column R may be left blank for this aircraft. The Manipulate_Fleet_Data macro (see Step 5) will assume that the AFHRS for this aircraft is equal to the log average AFHRS of all cracked aircraft for which the AFHRS are known.* *Entry of Aircraft ID is optional. However, entry of this information is useful when identifying outliers and whether there are reasons for them to be outliers (eg test aircraft). If data from multiple locations is being used, it is desirable to include a reference to both aircraft and location in the Aircraft ID field for the same reasons.* *Up to 9,985 data points may be entered. Additional data points will be ignored, but may be included if appropriate changes are made to the Manipulate_Fleet_Data macro.* *Warning: Ensure all pre-existing data is cleared from cells Q15 to S10000 to prevent incorrect data being included in the analysis.*	**Q15 to S10000**

Step	Description / *Guidance*	Cell Ref.
5.	**Click on the 'Calculate' Command Button.** *Click on the 'Calculate' command button at cell D14 to run a macro that manipulates the Fleet 'A' data points into a form suitable for analysis.* *This macro is named 'Manipulate_Fleet_Data' and can also be run using the 'Ctrl + s' shortcut key.* *The last part of this macro is identical to the 'Solve_for_Mean_AFHRS' macro (Refer step 8). Therefore, if an Assumed Standard Deviation has already been entered (step 7), the mean AFHRS for Fleet 'A' based upon the Assumed Standard Deviation will be calculated.*	**D14**
6.	**Qualitatively review and revise Fleet 'A' data points** *Review the Fleet 'A' data points and the fitted trendline presented in the chart window. Identify data points that are outliers and may be adversely skewing the trendline. Trendlines generated by Excel tend to be overly influenced by the data points at either end of the data series. Hence, if these data points are outliers then Excel will adversely skew the trendline toward these points.* *The effect of possible outliers may be assessed by removing (deleting) the trendline data pertaining to these outliers (located in cells X15 to Y10000). Cells will be shaded red where trendline data has been removed. This data may be reinstated by using the fill up or fill down tools in Excel (from the cells above or below the deleted data points). There is no need to perform step 5 again when these trendline data points are deleted and reinstated.* *If it is determined that identified outliers are adversely skewing the trendline, then the trendline data points pertaining to these outliers should stay deleted. If the aircraft is considered or assumed to be representative of the fleet, then the data should remain in the fleet data set (thus having an impact upon the ranked cumulative probabilities).* *However, if an aircraft is identified as being unrepresentative of the fleet, then it should be removed from the data set completely and the process restarted from step 5 (thus recalculating the ranked cumulative probabilities). An example of such an outlier may be a single flight test aircraft operated differently to the rest of the fleet and cracking at AFHRS significantly lower than the rest of the fleet.*	**X15 to Y10000** Q15 to S10000
7.	**Enter an Assumed Standard Deviation for the Lognormal Cumulative Probability of Cracking Distribution** *This assumed standard deviation may be based on the fleet data trendline, historical data, the standard deviation for similar locations, or other sources.*	**D8**
8.	**Click on the 'Calculate' Command Button.** *Click on the 'Calculate' command button at cell D22 to run a macro that calculates the mean AFHRS for Fleet 'A' based upon the Assumed Standard Deviation. The mean AFHRS is the AFHRS corresponding to a cumulative probability of 0.5 (50%).* *This macro is named 'Solve_for_Mean_AFHRS' and can also be run using the 'Ctrl + m' shortcut key.* *This macro utilises the Excel Solver add-in. Therefore, if no solution can be found (indicated by "#NUM!" in cell D21), then the initial starting value for the mean AFHRS may need to be adjusted. The initial starting value for the mean AFHRS calculation is stored in cell J2. Adjust this initial value closer to an estimate of the mean AFHRS and click on the 'Calculate' command button to rerun the macro.* *If an Assumed Standard Deviation is entered prior to Step 5, then this step is not required as the 'Solve_for_Mean_AFHRS' macro (Step 5) solves for the mean AFHRS.*	**D22**

Step	Description / *Guidance*	Cell Ref.
9.	**Enter a Severity Factor.** *Enter the severity factor that relates the usage severity of Fleet 'A' and Fleet 'B'. This severity factor is used to calculate the mean AFHRS for Fleet 'B' from the mean AFHRS for Fleet 'A'. The severity factor is equal to the usage severity of Fleet 'A' divided by the usage severity of Fleet 'B'.* *If no severity factor is entered, a severity factor of 1 is assumed (ie Fleet 'A' and Fleet 'B' have the same usage severity). This option is useful if the cumulative probability of failure of Fleet 'A' is required (it will be the same as Fleet 'B' when the severity factor is equal to 1).* *Further information regarding calculation and use of the severity factor is presented in Chapter 4.10.*	**D25**
10.	**Enter a Name for Fleet "B" (Optional)** *If desired, enter a name for Fleet 'B'. Ideally, use short names or acronyms to avoid errors in the presentation of the data (due to overlapping or wrapping of data text). If no name is entered, the fleet will be referred to as Fleet 'B'.*	**D28**
11.	**Enter the Number of AFHRS for Crack Growth from Cracking to Failure. (Optional)** *Enter the number of AFHRS required for cracking to progress to failure under Fleet 'B' usage.* *These AFHRS may be calculated using a crack growth curve for the location under consideration, with failure assumed to occur at the critical crack size (based on residual strength). The initial crack size is equal to the crack size on which the cracking data was derived.* *Refer to Chapter 4 for further guidance regarding determination of the initial crack size.*	**D31**
12.	**Enter the Cumulative Probability of Cracking / Failure for which the Corresponding AFHRS are Required. (Optional)** *If "Probability of Cracking" was selected in Step 2, then enter the cumulative probability of cracking for which the corresponding AFHRS are required. For example, enter "0.001" to calculate the AFHRS at which the cumulative probability of cracking is equal to 1 in 1000. If "Probability of Failure" was selected in Step 2, then the AFHRS at which the cumulative probability of failure is equal to the cell D36 will be calculated. These calculations are based on the Fleet 'B' cumulative probability of crack / failure.*	**D36**
13.	**Enter the AFHRS for which the Cumulative Probability of Cracking / Failure is Required. (Optional)** *If "Probability of Cracking" was selected in Step 2, then enter the AFHRS at which the cumulative probability of cracking is required. For example, enter "10,000" to calculate the cumulative probability of cracking at 10,000 AFHRS. If "Probability of Failure" was selected in Step 2, then the cumulative probability of failure will be calculated for the entered AFHRS. These calculations are based on the Fleet 'B' cumulative probability of crack / failure.*	**D40**
14.	**Enter the Fatigue Index (FI) Rate for Fleet 'B' (Optional).** *Enter the average Fatigue Index (FI) accrual rate for Fleet 'B' in terms of FI per 1,000 AFHRS.* *This number is used to calculate an approximate FI for the inspection threshold, assuming inspections are to begin at the AFHRS calculated in Step 12 (at the user specified cumulative probability of cracking).*	**D44**

Step	Description / *Guidance*	Cell Ref.
15.	**Revise Entries as Required (Optional).** *Data entered in previous steps may be revised at any time.* *If Fleet 'A' data points are revised, then the revised data points will have to be manipulated by performing Step 8 again.* *If assumed standard deviation (Step 7) is revised, the mean AFHRS of Fleet 'A' will need to be manually recalculated by performing Step 8 again).* *Revision of all other entries will be automatically incorporated.*	**See Steps 1 to 14**

7.5 Output Data Presented

All relevant output data is presented in the input/output data window and chart window (refer to Figure 11).

In addition to the input data, the input/output data window contains the output data presented in Table 3.

Table 3: Output Data Presented in Input/Output Data Window

Description of Output Data	Cell Location
Number of Cracked Aircraft in Fleet 'A'.	D16
Number of Aircraft in Fleet 'A'.	D17
Percentage of aircraft cracked in Fleet 'A'.	D18
Average AFHRS of Cracked Aircraft in Fleet 'A'	D20
AFHRS corresponding to a cumulative probability of cracking in Fleet 'A' of 0.5 (50%).	D21
AFHRS corresponding to a cumulative probability of cracking in Fleet 'B' of 0.5 (50%).	D30
AFHRS corresponding to a cumulative probability of failure in Fleet 'B' of 0.5 (50%).	D32
AFHRS corresponding to the user defined cumulative probability of cracking / failure.	D37
Cumulative probability of cracking / failure corresponding to the user defined AFHRS.	D41
Fatigue Index (FI) for the inspection threshold, assuming inspections are to begin at the AFHRS corresponding to the user defined cumulative probability of cracking (cell D36).	D45

The chart window presents the information presented in Table 4.

Table 4: Output Data Presented in Chart Window

Output Data	Description
Title (As entered in cell C3)	Chart Heading
Probability of cracking or probability of failure (as selected by the user in cell C5)	Chart Sub-heading
Mean and logSD of cumulative probability distribution trendline fitted to Fleet 'A' data points along with the goodness of fit (R-squared value).	Text at Top of Chart under Headings
Mean and assumed logSD for Fleet 'A' cumulative probability of cracking distribution.	Text at Top of Chart under Headings
Fleet 'A' data points	Black Crosses
Fleet 'A' data points used to generate trendline	Black Unfilled Circles
Data Point Representing Average AFHRS of Fleet 'A' Cracked Aircraft	Black Triangle
Cumulative probability distribution trendline fitted to Fleet 'A' data points.	Black Solid Line
Cumulative probability of cracking distribution for Fleet 'A'.	Large Brown Dashed Line
Cumulative probability of cracking distribution for Fleet 'B'.	Large Brown Dashed-Dotted Line
Cumulative probability of failure distribution for Fleet 'B'.	Small Brown Dashed Line
AFHRS corresponding to the user defined cumulative probability of cracking / failure.	Red Solid Line
Cumulative probability of cracking / failure corresponding to the user defined AFHRS.	Blue Solid Line

The calculations and macro operations that form the basis of this template are described in Appendix A.

8. Template Troubleshooting

8.1 Introduction

This Chapter provides basic troubleshooting information on common issues arising from use of the Excel cumulative probability chart templates.

8.2 Macros Will Not Run

These templates use macros to perform calculations and manipulate data. High security settings in Excel prevent macros from running. It may therefore be necessary to adjust the security settings in Excel to enable macros to be run.

Both macros used in the Excel templates rely upon the Excel Solver add-in being available to Excel. To confirm that the Solver add-in is available, click on Add-Ins… under the Tools menu and check the Solver Add-In checkbox if it is not already checked.

As the Solver function is being run from the macro, there also needs to be a reference to the Solver Add-In within Visual Basic (VBA). If the Solver Add-In is not referenced, the following error will appear when the macros are run: "Sub or Function not defined". If this error appears, perform the following steps to reference the Solver Add-In within VBA:

- Click on Solver in the Tools menu to display the Solver Parameters dialog box within Excel.
- Open the VBA editor (Alt+F11).
- Click on References in the Tools menu to display the available references.
- Check the SOLVER checkbox.

8.3 Mean AFHRS for Fleet 'A' are Not Calculated

If the mean AFHRS for Fleet 'A' are not being calculated even though the macros appear to be running, it is possible the solver could not converge on a solution (A #NUM! error may appear in cell D21).

In most cases, this problem can be fixed by adjusting the initial estimate for the mean AFHRS (contained in cell J2) and rerunning the macro. When adjusting the initial estimate of mean AFHRS, shifting it closer to the estimated mean AFHRS is more likely to ensure convergence on a solution.

If the above solution does not work, it may be due to the assumed value for the standard deviation. Assume a slightly different value and rerun the macro.

For advanced uses, it is possible to run the solver directly from within Excel without the macro and adjusting the options within the solver. In both templates, the mean AFHRS for Fleet 'A' was found by using the solver to change cell K2 to set the value of cell K10 to zero.

8.4 Data Points Do Not Fall on a Relatively Straight Line

Possible reasons for the Fleet 'A' data points not falling on a relatively straight line when plotted on the cumulative probability chart include the following:

- The fleet data does not follow a lognormal distribution.
- The fleet data hasn't been normalised to a nominal baseline crack length.
- The cracked aircraft in the fleet have not been subjected to similar usage.
- Cracking at that location is not strongly dependent upon AFHRS. Other possible metrics for cracking which may have a stronger relationship are landings and pressurisations.

8.5 The Average Data Point Does Not Fall on the Trendline

The average data point will not necessarily fall on the trendline as the trendline is influenced by the scatter of the constituent data points. However, if the trendline accurately represents the distribution, the difference should be relatively small.

8.6 Chart and Text in Chart Window is Not Pretty Enough

Depending upon the input, the chart and text in the chart window may not be presented as expected due to text wrapping, overlaps or the positioning of the cumulative probability distributions.

The format of the chart and all text boxes in the chart window can be easily modified by the user to suit their own needs.

8.7 Excel NORMDIST Error

If all aircraft are cracked, the highest rank will be 1.0. This should be manually adjusted to 0.9999 to remove this error, as Excel cannot handle the probability of 1.0 within the probability analysis.

8.8 In Detailed Data Spreadsheet, the aircraft listed in Cell D17 is different to those entered

The count feature counts the number of entires in Column Q titled Aircraft ID. Even though the aircraft ID number, or BuNo number is not important (although it assists in determining possible outliers), an entry, even if it is a dummy number, must be entered to enable the count function to work correctly.

9. Conclusion

Probabilistic analysis of fleet inspection or test data can be used to ensure that aircraft structural integrity is maintained to an acceptable level. By employing these methods, the risk of cracking (or failure) at specific times in an aircraft's life can be determined. This analysis can then be used to determine the management strategy for a fleet of aircraft.

A Microsoft Excel template has been developed which allows the determination of lognormal probability distributions of cracking (or failure) from test article or fleet aircraft data. The template has been designed to enable easy application of the probabilistic analysis to any fleet of aircraft. A comprehensive user guide to the template has also been provided which includes assumptions, layout, data entry, output data and the underlying calculations. This template may be used to quickly and simply assess the life of aircraft structures using a probabilistic approach.

This template was used in the development of the RAAF P-3C Structural Management Plan. The template enabled the analysis of the significant amount of P-3C inspection data available, and the risk of cracking at each critical location on RAAF aircraft to be determined.

10. References

1. Anon, "Defence Standard 00-970, Design and Airworthiness Requirements for Service Aircraft", Issue 5, 31 January 2007.

2. E.K. Walker, "Forecasting the Risk of Fleetwide Fatigue Cracking in Aircraft Structure – A Workshop Prepared for the RAAF and AMRL", May 1996.

3. DSTO-TR-1947, "Australian P-3 Structural Management Plan", David Mongru, Kai Maxfield, Brendan Murtagh, Emilio Matricciani, Len Meadows and Philip Jackson, December 2006.

4. DSTO-TR-2186, "P-3 Lower Wing Surface Panel Spanwise Splices Revised Approach to Set Inspection Thresholds, Emilio Matricciani, Kai Maxfield and Len Meadows, October 2008.

5. P. Jackson, "DSTO Review of P-3 Safe Life Extension Proposal", DSTO B2/243 Pt 1 Item 9 Annex A: "Calculation of Probability of Cracking of Inboard Nacelle Dome Nut Holes", 17 February 2003.

Appendix A: Worksheet Calculations and Macro Operations

A.1. Cumulative Probability Worksheet for Summary Data

This section presents a step by step guide to the significant calculations and macro operations within the worksheet. It is assumed that all required data has been input. Simple calculations and operations related to presentation of the data on the worksheet should be self evident to the user and are not detailed here. Information regarding Excel functions used in these calculations and operations should be obtained from Excel help or other authoritative sources.

Step	Description / *Guidance*	Cells Affected
1.	**Calculate Cumulative Probability of Cracking of the Average Cracked Aircraft in Fleet 'A'**	**K6**
	By default, the cumulative probability of cracking of the average cracked aircraft is taken as the percentage of aircraft cracked (if entered in cell D13). If this information is not entered, the cumulative probability of cracking of the average cracked aircraft is calculated using the number of aircraft cracked and the number of aircraft in the fleet (entered in cells D16 & D17). If the number of aircraft cracked is equal to the number of aircraft in the fleet, the cumulative probability of cracking is assumed to equal 0.5 (50%). *Refer to Chapter 3 for important information regarding calculation of the cumulative probability of cracking of the average cracked aircraft.*	

Step	Description / *Guidance*	Cells Affected
2.	**Calculate the Mean of the Cumulative Probability of Cracking Distribution in Fleet 'A'** *The mean of the cumulative probability of cracking distribution in Fleet 'A' is stored in cell K2. If 'Solve_for_Mean_AFHRS' macro has not been run, then the value in this cell may not be the correct mean.* *The cumulative probability of cracking of the average cracked aircraft in Fleet 'A' is calculated in cell K9 using the mean (stored in cell K2), the assumed standard deviation (entered in cell D8), and the average AFHRS of the cracked aircraft (entered in cell D20). The Excel function NORMDIST is used to perform this calculation and requires the use of LOG(AFHRS) rather than AFHRS.* *If the mean is correct, this cumulative probability of cracking (cell K9) should equal the cumulative probability of cracking calculated using the input data in step 1 (cell K6). The difference between the two is presented in cell K10. If the difference is greater than the allowable difference stored in cell K11, then the mean stored in cell K2 is incorrect. The'Solve_for_Mean_AFHRS' macro must then be run to solve for the mean of the cumulative probability of cracking distribution. The 'Calculate' command button at cell D22 will run this macro.* *The 'Solve_for_Mean_AFHRS' macro calls on the Excel solver to change the mean (cell K2) such that the difference in cumulative probability of cracking (cell K10) is zero. While the allowable difference (stored in cell K11) is not used during the solve, it is used to indicate to the user when mean is incorrect (the cells presenting the mean cumulative probability of cracking distribution for Fleet 'A' and 'B' in the input/output window are highlighted pink).* *The default allowable difference (stored in cell K11) of 1e-4 was based on experience of a suitable tolerance achievable by the solver.* *Before running the solver, the 'Solve_for_Mean_AFHRS' macro sets the mean (cell K2) equal to an initial value (stored in cell J2) in case the solver diverged from a solution on the previous run (resulting in an error result in cell K2). The optimum value for this initial mean (cell J2) will depend upon the input data, but a value close to the expected mean will ensure a higher probability of convergence by the solver.*	**K2, K9, K10, K11**
3.	**Calculate the Mean of the Cumulative Probability of Cracking Distribution in Fleet 'B'** *This mean of the cumulative probability of cracking distribution in Fleet 'B' is stored in cell L2, and calculated by multiplying the mean of the cumulative probability of cracking distribution in Fleet 'A' by the severity factor (entered in cell D25).* *Note the standard deviation of Fleet 'B' is assumed to be the same as Fleet 'A'.*	**L2**
4.	**Calculate the Mean of the Cumulative Probability of Failure Distribution in Fleet 'B'** *The mean of the cumulative probability of failure distribution in Fleet 'B' is stored in cell M2, and calculated by adding the crack growth AFHRS from cracking to failure (entered in cell D31) to the mean of the cumulative probability of cracking distribution in Fleet 'B'.* *Note that due to the nature of this operation, the cumulative probability of failure distribution is no longer a lognormal distribution, hence a standard deviation is not applicable. However, it is still possible to plot this distribution on the lognormal cumulative probability chart.*	**M2**

Step	Description / *Guidance*	Cells Affected
5.	**Calculate How Many Standard Deviations are Between the Average AFHRS of the Cracked Aircraft in Fleet 'A' and the Mean of the Cumulative Probability of Cracking Distribution in Fleet 'A'** *The number of standard deviations between the mean and any given point on the cumulative probability of cracking distribution is equal to:* $\text{No. of Standard Deviations} = \dfrac{\log(\text{AFHRS})-\log(\text{Mean AFHRS})}{\log \text{SD}}$	**L6**
6.	**Calculate the AFHRS Corresponding to the User Defined Cumulative Probability of Cracking (or Failure) in Fleet 'B'** *The AFHRS corresponding to the user defined cumulative probability of cracking (or failure) in Fleet 'B' (entered in cell D36) is calculated in cell N24 using the mean of the cumulative probability of cracking (or failure) distribution (stored in cell L2), the assumed standard deviation (entered in cell D8), and the user defined cumulative probability of cracking (or failure). The Excel function NORMINV is used to perform this calculation and requires the use of LOG(AFHRS) rather than AFHRS.* *The calculations described above are based on the cumulative probability of cracking distribution. Therefore, if the user has selected to present information for the cumulative probability of cracking (selection in cell C5), then no adjustment is made to the calculations. However, if the user has selected to present information for the cumulative probability of failure, then the crack growth from cracking to failure (entered in cell D31) must be added to the calculated AFHRS.* *Note the standard deviation of Fleet 'B' is assumed to be the same as Fleet 'A'.*	**N24**
7.	**Calculate the Cumulative Probability of Cracking or Failure Corresponding to the User Defined AFHRS in Fleet 'B'** *The cumulative probability of cracking (or failure) corresponding to the user defined AFHRS in Fleet 'B' (entered in cell D40) is calculated in cell M25 using the mean of the cumulative probability of cracking (or failure) distribution (stored in cell L2), the assumed standard deviation (entered in cell D8), and the user defined AFHRS. The Excel function NORMDIST is used to perform this calculation and requires the use of LOG(AFHRS) rather than AFHRS.* *The calculations described above are based on the cumulative probability of cracking distribution. Therefore, if the user has selected to present information for the cumulative probability of cracking (selection in cell C5), then no adjustment is made to the calculations. However, if the user has selected to present information for the cumulative probability of failure, then the crack growth from cracking to failure (entered in cell D31) must be subtracted from the defined AFHRS prior to the calculation.* *Note the standard deviation of Fleet 'B' is assumed to be the same as Fleet 'A'.*	**M25**

Step	Description / *Guidance*	Cells Affected
8.	**Generate Plot Data for Cumulative Probability Chart** *It is desired to plot data on a lognormal cumulative probability chart, with a log scale for the AFHRS (x-axis) and the unique cumulative probability log scale for the y-axis (refer Chapter 3).* *Due to the unique cumulative probability log scale, Excel cannot automatically generate this type of chart. However, there is a one to one relationship between cumulative probability and the number of standard deviations from the mean (as shown in Figure 6). As the number of standard deviations from the mean is linear along the y-axis, the lognormal cumulative probability chart can be emulated in Excel by plotting AFHRS vs the number of standard deviations from the mean.* *To maintain the look of the lognormal cumulative probability chart, it is necessary to plot "artificial" cumulative probability gridlines on the chart. The data for these artificial gridlines is located in cells Q1 to T292. The data is presented in pairs, representing the gridline endpoints. The AFHRS at the minimum and maximum endpoints are defined by the user in cells S2 and S3. These minimum and maximum values should match the minimum and maximum values for the AFHRS scale (x-axis) in the cumulative probability chart, otherwise the data labels for selected cumulative probability gridlines (used as an artificial cumulative probability scale) may not be presented correctly. The number of standard deviations from the mean was calculated from the cumulative probability for each gridline via the Excel NORMINV function with the mean and standard deviation assumed to equal 1 (the values of mean and standard deviation used are irrelevant to the calculation).* *The plot data for the cumulative probability of cracking distribution for Fleet 'A' is presented in cells I13 to N16. The minimum and maximum AFHRS defining the endpoints of the distribution (straight line on chart) are the same as assumed for the "artificial" cumulative probability gridlines. The cumulative probability of cracking for these endpoints was calculated via the Excel NORMDIST function and the mean and standard deviation of the Fleet 'A' cumulative probability of cracking distribution (stored in cells K2 and K4). The number of standard deviations from the mean was calculated in the same manner as described in step 5.* *The plot data for the cumulative probability of cracking distribution for Fleet 'B' is presented in cells I18 to N21. This data was calculated in the same manner as for the Fleet 'A' cumulative probability of cracking distribution, using the Fleet 'B' data as appropriate.* *The plot data for the cumulative probability of failure distribution for Fleet 'B' is presented in cells I42 to N64. In this case, the AFHRS corresponding to the cumulative probability of cracking distribution for Fleet 'B' (cells I43 to I64) were calculated for a range of values for the number of standard deviations from the mean (cells N43 to N64). The AFHRS corresponding to the cumulative probability of failure distribution were then determined by adding on the AFHRS for crack growth from cracking to failure.* *The plot data for the intercept lines used on the chart to pinpoint the user defined cumulative probability of cracking / failure in Fleet 'B' (and the corresponding AFHRS) are presented in cells I29 to N34. The plot data was generated for the intercept point stored in cells M24 and N24.* *The plot data for the intercept lines used on the chart to pinpoint the user defined AFHRS in Fleet 'B' (and the corresponding cumulative probability of cracking / failure) are presented in cells I35 to N40. The plot data was generated for the intercept point stored in cells M25 and N25.* *Chart series titles are stored in cells I66 to N71.* *Chart text summarising the user defined intercepts are stored in cells K26 and K27.*	**Q1 to T292**

Step	Description / *Guidance*	Cells Affected
	Where appropriate, the source data of the chart titles, labels and text have been designed to substitute in the user defined names for Fleet 'A' and Fleet 'B' if they are entered. Otherwise Fleet 'A' and Fleet 'B' will be used to describe the two fleets. *Where appropriate, the source data of the chart plot data, titles, labels and text has been designed to appear 'Null' if the appropriate input data has not been entered or an error is detected (ie cumulative probability distributions have not been calculated by macro). This was intended to prevent the chart displaying incorrect information.*	

A.2. Cumulative Probability Worksheet for Detailed Data

This section presents a step by step guide to the significant calculations and macro operations within the worksheet. It is assumed that all required data has been input. Simple calculations and operations related to presentation of the data on the worksheet should be self evident to the user and are not detailed here. Information regarding Excel functions used in these calculations and operations should be obtained from Excel help or other authoritative sources.

This worksheet is almost identical to the 'Prob Chart for Summary Data' worksheet except that there have been 11 columns inserted to store the data pertaining to the Fleet 'A' data (columns Q to AA). There have also been some minor changes in the input / output and chart window to reflect the use of fleet detailed cracking data rather than fleet summary cracking data.

Therefore, this section only presents worksheet calculations and macro operations that differ from that presented for the 'Prob Chart for Summary Data' worksheet in Appendix A.1.

Step	Description / *Guidance*	Cells Affected
1.	**Differences in Input / Output Window When Compared to the 'Prob Chart for Summary Data' Worksheet** *When compared to the 'Prob Chart for Summary Data' worksheet:* *a. the option of entering the percentage of aircraft cracked (originally in cell D13) has been removed,* *b. the number of aircraft cracked (cell D16) and the number of aircraft in the fleet (D17) are now automatically entered from cells U1 and S1 respectively (counted from the Fleet 'A' data entered in cells Q15 to S10000),* *c. the average AFHRS of the cracked aircraft in Fleet 'A' is now automatically entered from cell S3 (calculated from AFHRS of cracked aircraft within the Fleet 'A' data entered in cells Q15 to S10000),* *d. A command box has been inserted at cell D14 to run the 'Manipulate_Fleet_Data' macro.*	**D13, D16, D17, D20**

Step	Description / *Guidance*	Cells Affected
2.	**Calculate the Number of Aircraft in Fleet 'A' and the Number of Aircraft Cracked in Fleet 'A'** *The number of aircraft in Fleet 'A' (stored in cell S1) is calculated from the Fleet 'A' detailed data by counting the number of aircraft tail number entries in cells Q15 to Q10000. This is why an entry is required in this column for every aircraft in the fleet, even if the entry is a dummy number.* *The number of cracked aircraft in Fleet 'A' (stored in cell U1) is calculated from the Fleet 'A' detailed data by counting the number of entries in cells S15 to S10000. The type of entry designating a crack is unimportant, as long as it is consistent (for later sorting purposes).*	**S1 & U1**
3.	**Manipulate the Fleet 'A' Detailed Data Points Into a Form Suitable for Analysis** *The detailed data points for Fleet 'A' that have been entered in cells Q15 to S10000 may or may not be ordered with respect to AFHRS and whether or not the aircraft is cracked or not.* *The 'Manipulate_Fleet_Data' macro sorts the detailed data points with respect to whether they are cracked or not (cracked aircraft at the top), then sorts by AFHRS (ascending order).* *Prior to sorting the Fleet 'A' detailed data, the 'Manipulate_Fleet_Data' macro performs the following tasks:* *a. Calculation of the last row of the fleet data points and calculation of the last row of cracked data points (after later sorting).* *b. Calculation of the logarithmic average AFHRS of the fleet and store value in cell S2. Aircraft missing AFHRS are skipped in this calculation.* *c. Calculation of the logarithmic average AFHRS of the cracked aircraft and store value in cell S3. Aircraft missing AFHRS are skipped in this calculation.* *d. Calculation of the logarithm AFHRS of each cracked aircraft and store values in cells T15 to T10000. If the AFHRS of a cracked aircraft are missing, its AFHRS are assumed to equal that of the logarithmic average AFHRS of the cracked aircraft.* *Once the Fleet 'A' detailed data is sorted, the formulas in cells U15 to AA15 are filled down for all of the cracked aircraft. These cells contain:* *a. U15: AFHRS for the cracked aircraft, substituting the logarithmic average AFHRS of the cracked fleet if the AFHRS are missing.* *b. V15: The ranked cumulative probability of cracking, calculated as detailed in Chapter 3.* *c. W15: The number of standard deviations from the mean for the data point* *d. X15 & Y15: The AFHRS and number of standard deviations from the mean for the data point for use in the trendline fitting. These values are linked to cells U15 and W15 respectively.* *d. Z15 & AA15: The AFHRS and number of standard deviations from the mean for the data point for display in the cumulative probability chart. These values are linked to cells U15 and W15 respectively.* *The final operation in this macro is to calculate the mean of the cumulative probability of cracking distribution. This operation is identical to that performed by the 'Solve_for_Mean_AFHRS' macro described in step 2 in Appendix A.1.*	**Q15 to S10000**

Step	Description / *Guidance*	Cells Affected
4.	**Calculate the Trendline for the Fitted Data** *The slope and Y-intercept for a trendline fitted to the data point data in cells Y15 to Y10000 and T15 to T10000 are calculated using the Excel SOPE and INTERCEPT functions and stored in cells S5 and S7 respectively. The Y-intercept is presented in terms of number of standard deviations from the mean.* *The standard deviation for the trendline is calculated from the slope of the trendline (1/slope) and stored in cell S6.* *The logarithm of the mean AFHRS of the trendline (cumulative probability equal to 0.5) is stored in cell S8 and is calculated using the slope and the Y-intercept of the trendline, as follows:* *Log(Mean AFHRS) =Y-intercept / Slope* *The mean AFHRS of the trendline is calculated from the log (Mean AFHRS) and is stored in cell S9.* *The Goodness of Fit of the trendline is calculated using the Excel RSQ function and the cells Y15 to Y10000 and T15 to T10000 and is stored in cell S10.*	**S5 to S10**

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Probability, Aircraft Structures, Fatigue Life, Software

19. ABSTRACT

To ensure aircraft structural integrity is maintained to an acceptable level, probabilistic approaches may be used to calculate the risk of cracking (or failure) over the life of the aircraft or fleet. One such risk analysis technique employs a lognormal probability distribution to model the likelihood of cracking (or failure) in the fleet with respect to hours. This technique was programmed into Microsoft Excel to create a simple and easy to use template. An outline of the theory behind the probabilistic approach is provided as well as a comprehensive user guide to the template. This template allows the quick and simple determination of probability distributions of cracking (or failure) which may be used to assess the life of aircraft structures.